1. Use the Black-Scholes formula to find the value of a call option on the following stock:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to expiration</td>
<td>6 months</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>50% per year</td>
</tr>
<tr>
<td>Exercise Price</td>
<td>$50</td>
</tr>
<tr>
<td>Stock Price</td>
<td>$50</td>
</tr>
<tr>
<td>Interest Rate (Per annum)</td>
<td>10%</td>
</tr>
</tbody>
</table>

\[
C_0 = S_0 N(d_1) - X e^{-rt} N(d_2)
\]

\[
d_1 = \frac{\ln \left( \frac{S_0}{X} \right) + \left( r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} = \frac{\ln \left( \frac{50}{50} \right) + (0.1 + 0.5^2/2)0.5}{0.5\sqrt{0.5}} = 0.6364
\]

\[
d_2 = d_1 - \sigma \sqrt{T} = 0.6364 - 0.3536 = 0.2828
\]

\[
C_0 = 50N(0.6364) - 50e^{-0.1\times0.5}N(0.2828) = 50\times0.7377 - 50\times0.9512\times0.6113 = 7.81
\]

2. Find the value of put option on the stock in the previous problem with the same information above (Hint: there are two ways of calculating such value).

Sol 1: Using Put-Call Parity

\[
P = C + PV(X) - S_0
\]

\[
P = C + X e^{-rt} - S_0
\]

\[
P = 7.81 + 50 e^{-0.1\times0.5} - 50 = 5.37
\]

Sol 2: Using Put option formula

\[
P = X e^{-rt} (1 - N(d_2)) - S_0 (1 - N(d_1))
\]

\[
P = 50e^{-0.1\times0.5}(1-0.6113) - 50(1-0.7377) = 5.37
\]

which is equal to the above

3. You would like to be holding a protective put position on the stock of XYZ Company to lock in a guaranteed minimum value of $100 at year-end. XYZ currently sells for $100. Over the next year, the stock price will either increase by 10% or decrease by 10%. The T-Bill rate is 5.0%. Suppose the desired put option were traded. How much would it cost to purchase?
\[ S_0 = 100, \ X = 100, \ u = 1.1, \ d = 0.9, \ r_f = 0.05 \]

**Binomial for call option**

**Stock**

- 110
- 90

**Call Option**

- C
- 10
- 0

\[ u = 1.1, \ d = 0.9 \]

**Alternative Portfolio**

**Buy 1 share of stock at $100**

**Borrow $85.714 at 5% interest rate**

\[ \text{Net Outlay} = \$14.286 \]

**Payoffs**

\[
\begin{array}{c|cc}
\text{Stock value} & 90 & 110 \\
\hline
\text{Repay Loan (after 1 Year)} & -90 & -90 \\
\text{Net Payoff (after 1 Year)} & 0 & 20 \\
\end{array}
\]

**Put-Call parity**

\[ P = C + X e^{-r t} - S_0 = 7.143 + 100 e^{(0.05 \times 1)} - 100 \]

\[ P = \$2.27 \]
4. What is the elasticity of a call option currently selling for $4 with the exercise price of $120 and a hedge ratio of 0.4 if the stock price is currently at $122?

Elasticity = percentage change in option price given 1% change in underlying asset price

\[ \frac{\Delta C}{C} = \frac{\Delta S}{S} \]

For $1 dollar change in stock price

\[ \Delta S/S = 1/122 = 0.819\% \]

\[ \Delta C = H \times \Delta S = 0.4 \times 1 = 0.4 \]

Therefore \[ \Delta C/C = 0.4/4 = 10\% \]

Elasticity = \[ 10\% / 0.819\% = 12.21 \]

High elasticity because options are highly leveraged

5. These three put options all are written on the same stock. One has a delta of -0.9, one has a delta of -0.5, and one a delta of -0.1. Assign deltas to the three puts by filling in the table below.

<table>
<thead>
<tr>
<th>Put</th>
<th>X</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15.5</td>
<td>-0.9</td>
</tr>
<tr>
<td>B</td>
<td>25.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>C</td>
<td>35.5</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

6. According to the Black-Scholes formula, what will be the value of the hedge ratio of a call option as the stock price becomes infinitely large? Explain Briefly.

Black-Scholes Hedge Ratio \( = N(d_1) \)
As \( S_0 \) approach infinity, \( d_1 \) also converge to infinity

\[ \lim_{d_1 \to \infty} N(d_1) = 1 \quad \Rightarrow \quad \text{Black-Scholes Hedge Ratio} = N(d_1) = 1 \]

for deep-in-the-money call option, its payoff will closely track stock value payoff (slope=1).