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Including Geological Uncertainty and Economic Analysis in a Rapid Simulation of Hydrocarbon Exploration

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Manly's approximation method has been applied to hydrocarbon discovery process modeling in order to approximate the expected value and the standard deviation of the total discovery volume for a given exploratory effort. A major benefit of this method is that it allows the model to run much faster than the regular simulation method, yet it gives accurate results. This paper extends the benefit of Manly's approach by allowing the approximation method to incorporate the uncertainties in geological parameters that drive Manly's approximation, in order to provide an approximation of the complete distribution of total discovery volume that can result from exploration activity. In addition, it allows the model to include economic parameters into an evaluation of the economic worth of the results of exploration activity, producing distributions of net present value within a short period of time. The offshore Nova Scotia Shelf Basin is selected for demonstrating the methodology.

KEY WORDS: Probabilistic model; Manly's approximation; field-size distribution; number of fields; Nova Scotia Shelf.

INTRODUCTION

There are tremendous financial risks attached to exploration for hydrocarbons in frontier regions because of the high uncertainties in geological and economic factors. Therefore, it is essential for a company to try to assess the potential results of future exploration before making a decision to explore any region. An inadequate appreciation of the uncertainties in geological and economic parameters can lead to misjudged policies, costly exploration failures, and other severe consequences for oil and gas companies.

To cope with these uncertainties, various techniques based on different approaches have been developed and are being used to forecast future hydrocarbon discoveries in an effort to reduce the risk. These approaches range from the basin and play level of analysis to continental aggregations and from detailed structural and process models to simple extrapolations and curve-fitting. They also range from geologic-based attempts to estimate the *in situ* resource base to the economic based estimates of supply (Power and Fuller, 1992). These techniques provide systems for evaluating the future discoveries of hydrocarbons and economic outcomes. However, some of them do not yield objective quantitative appraisals, because they involve an intuitive blending of geological qualities and subjective weighting qualities. Some do not allow for quantitative assessment of risk and uncertainty in hydrocarbon exploration and cannot account for other possible reserve levels and economic uncertainties.

A probabilistic model of the hydrocarbon discovery process has been accepted widely from the 1970s for evaluating the future discoveries of hydrocarbons because of its capability of incorporating specific geological, technological, and economic attributes of the process of exploration. A precursor of this probabilistic model was the work of Arps and Roberts (1958), who

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first introduced the notions that the probability of discovering a field of a given size is proportional to the number of undiscovered fields of that size and to the areal extent of each field. Kaufman, Balcer, and Kruyt (1975) and Barouch and Kaufman (1976) established the probabilistic model and used maximum likelihood techniques for the estimation of the field-size distribution from the information in the historical discovery record. Several researchers also contributed to probabilistic formulations of the hydrocarbon exploration and discovery process in estimating the hydrocarbon reserves and in exploration policy analysis (see for example, Smith, 1980; O'Carroll and Smith, 1980; Lee and Wang, 1983a, 1983b, 1985; Schuenemeyer and Drew, 1983; Drew, Attanasi, and Schuenemeyer, 1988; Rabinowitz, 1991; Power and Fuller, 1991; Power and Jewkes, 1991; and Bickel, Nair, and Wang, 1992). However, prolonged simulations have been required when using the probabilistic model to forecast the distribution of the volume of future discoveries because of a given exploratory effort.

In 1991, a new approach in discovery process modeling for estimating the future hydrocarbon discoveries was introduced. Fuller (1991) applied Manly's approximation method, which initially was suggested by Manly (1974) in the context of biometrics experiments, to forecast the means and standard deviations of future discovery volumes as functions of the number of discoveries, or in some situations, of the number of wells drilled (including dryholes). Manly's method assumes the same postulates as the probabilistic model of hydrocarbon discovery. By using this method, running the model required only a few seconds on a computer, in comparison with several hours for traditional extensive simulation. The approximation method also gave accurate results for specific data with the parent population of fields having a lognormal distribution, exponential distribution, Weibull distribution, and gamma distribution (Ninpong, Power, and Fuller, 1992). Later, the approximation method was simplified further by Fuller and Wang (1993) and it was used by Macdonald, Power, and Fuller (1994) in a regression approach to forecasting future discoveries.

However, the approximation of only the first two moments of the distribution of total discovery volume is insufficient to approximate the whole distribution, because the probability density function is highly asymmetric for small and large well numbers. To solve this problem, Chungcharoen (1994) and Chungcharoen and Fuller (1996) used the Manly approximations of mean and standard deviation, together with the calculated smallest and largest possible total discovery volumes at any well number, to set the parameters of a Beta distribution that then approximates the true distribution of total discovery volume for a given well number. Confidence intervals of the forecast, based on the Beta distributions, were constructed and compared to the confidence intervals of the forecast from the simulation distributions. Several data sets were selected to verify their methodology and sensitivity analyses were performed to show that the idea of using a family of Beta distributions is a robust approximation. One limitation of this earlier work is its assumption that the driving geological parameters (number of fields of various sizes) are known with certainty. Another limitation is the lack of a framework for economic analysis with the aid of the forecasts—distributions of present worth cannot be constructed easily when the discovery forecasts are for total volumes. Present worth evaluation requires a distinction between dryhole and discovery well costs (with other costs related to volumes), and disaggregation by year of discovery, for discounting.

We continue Chungcharoen and Fuller's (1996) work in this paper by incorporating the uncertainties in geologic information into the calculations to provide estimates of the distributions of total discovery volume that may be recovered as the exploration progresses and combining these estimates with economic information to provide a probabilistic evaluation of the economic worth of exploration activities. The results can give a decision-maker a better picture of the range of both reserve additions and economic potential of exploration efforts in the underlying region. Thus, this work provides a useful exploration and policy-planning tool for use by government agencies as well as by oil and gas companies.

BACKGROUND

Full mathematical explanations of the probabilistic model of the hydrocarbon discovery process and Manly's approximation method are given in Chungcharoen (1994) or Chungcharoen and Fuller (1996). Brief explanations of these approaches follow.

The probabilistic model relies on two postulates. First, the model portrays the discovery phenomenon as a sampling process without replacement (Barouch and Kaufman, 1976). Second, the probability of discovery of an individual field is proportional to the field size (Arps and Roberts, 1958), that is, the areal extent of the field. Consider an unexplored area: if the drilling locations are selected randomly, the probability of discovering a field of a certain size with the first exploratory well can be expressed as the ratio of the product of the size and number of that size to the area available for exploration in the underlying basin. In some applications of the model, one of the field-size classes represents dryholes, the area available for exploration includes all potential areas (including those that turn out to be dry), and the model tracks all exploratory wells (including dryholes). In other applications, all size classes represent actual hydrocarbon deposits, the total "exploration" area is just the sum of the areal extents of all deposits, and the model tracks only the discovery wells.

Based on evidence from the drilling histories of wells, researchers have determined that there is more success in discovering fields than the random drilling rule would propose (e.g., Arps and Roberts, 1958; Attanasi and others, 1981). Hence the "discovery efficiency" parameter is introduced into the probabilistic model. This parameter measures the magnification of the influence of areal extent on the probability with which a field is discovered. The model is modified by raising the hydrocarbon field's size value in the probability expression to the power of the discovery efficiency. As exploration progresses, the number of undiscovered fields in each size class is the difference between the original number in that size class (i.e., before exploration began) and the number of discoveries of that size to date. From these relations, a simulation of sampling without replacement process can be implemented. Using a relationship between the areal extents of fields and their hydrocarbon volumes permits estimation of the distribution of discovered volumes for a given number of wells drilled.

Manly's approximation method uses the same postulates as the probabilistic model. Interpreted in the hydrocarbon discovery setting, Manly's approximation gives the approximate means and variance–covariance matrices of the number of a field-size class remaining undiscovered at a particular discovery number. In brief, the approximate mean number of fields remaining undiscovered in size class k after i discoveries is calculated by using recursion relations as follows:

$$\mu_{ki} = \mu_{ki-1} - \theta_{ki-1} \tag{1}$$

where μ_{ki} is the approximate mean value after i discoveries for field size class k,

$$\theta_{ki-1} = \frac{S_k^\beta \mu_{ki-1}}{\mu_{i-1}^*} \tag{2}$$

and

$$\mu_{i-1}^* = \sum_{j=1}^{K} S_j^\beta \mu_{ji-1} \tag{3}$$

where S_k^β is an areal extent of field-size class k raised to the power of the discovery efficiency, β, and K is the number of possible field-size classes. It can be inferred that the mean value of the total number of discovered fields in size class k at the ith discovery is equal to the number of fields initially estimated in size class k, A_k, less the mean value of the number of fields in size class k remaining after the ith discovery. Hence, the mean value of the total volume discovered after i discoveries is estimated by multiplying the mean number of fields discovered in each size class by the volume for that class and summing across size classes,

$$\sum_{k=1}^{K} V_k (A_k - \mu_{ki}) \tag{4}$$

where V_k is the volume per field in class k. The estimated standard deviation is determined as the square root of the approximate variance of the total discovery volume and is estimated by

$$\left[\sum_{k=1}^{K} \sum_{l=1}^{K} V_k V_l C_{kli} \right]^{1/2} \tag{5}$$

where C_{kli} is the approximate covariance between the numbers of remaining fields of sizes k and l after i discoveries. Manly's method estimates the approximate covariance by another recursion relation (see, for more detail, Manly, 1974; Chungcharoen, 1994; and Chungcharoen and Fuller, 1996).

Based on careful but lengthy exact simulations (without relying on Manly's approximation), it is known that the probability density function of the total discovery volume is highly asymmetric (right-skewed) for small numbers of discovered fields. As the number of discovered fields increases, it becomes roughly normal in appearance. Later, when the number of fields gets close to being exhausted, the density function of the total discovery volume becomes asymmetric again, but left-skewed. For a particular number of discovered fields, i, the maximum total amount of hydrocarbons discovered cannot exceed the sum of the volumes of the i largest fields. In addition, the minimum total amount of hydrocarbons discovered cannot be less than the sum of the volumes of the i smallest fields.

Chungcharoen (1994) selected the Beta distribution to fit the distribution of the total discovery volume for a given discovery number, using the minimum and maximum volumes for the range of the Beta distribution, and the mean and standard deviation obtained from Manly's approximation to determine the values of the two shape parameters of the Beta distribution. The Beta distributions were compared to the results of the exact simulations by using histograms and Kolmogorov–Smirnov (K–S) goodness-of-fit tests. The Beta distributions were shown to provide a good fit to the distributions of the total discovery volume from the beginning to the end of the exploration. This methodology was applied successfully to three real data sets: the Nova Scotia Shelf, the Bistcho Play, and the Zama Play.

METHODOLOGY

Including Uncertainty in Geological Parameters in the Approximate Distribution of Total Discovery Volume

There are two major geological uncertainties that play a key role in the hydrocarbon discovery process: the field sizes and the number of possible fields. In order to describe the distribution of possible sizes of a field that may exist in an exploration play or basin, the field-size equation based on Roy (1975), Proctor and Taylor (1984), and Lee and Wang (1990), is considered. Distributions of variables, such as, field area and net pay, in the field-size equation are based on interpretations by geologists or comparative studies; therefore, the subjective opinion of experts is the basis of estimates. Various distributions of geological parameters are multiplied together using the Monte Carlo method, taking care to account for correlations, to produce the field-size distribution in the play or basin. The field-size distribution is conditional in the sense that we are interested in identifying the distribution of field sizes given that hydrocarbons in fact have been generated, migrated, trapped, and preserved in fields (Proctor and Taylor, 1984). Because the objective of this work is to demonstrate how to incorporate uncertainties into frequency-size distributions, we assume that the field-size distribution already has been obtained.

In addition to the uncertainty in field size, there also is uncertainty in the number of fields. Therefore, the number of possible fields is represented by a probability distribution. According to Proctor and Taylor (1984), the number of fields distribution tends to be a relatively difficult distribution to prepare as geologists consistently underestimate this number. There is a particular tendency to not recognize the large number of relatively small fields that are associated with most areas.

After determining the field-size distribution and estimating the number of possible fields distribution, we use the Monte Carlo approach to sample from the number of fields distribution, and then sample that number of times from the field-size distribution. Each frequency-size distribution (numbers of fields of various sizes) is constructed based on categorizing the sampled field data into size classes. The average volume in each size class is determined when all size classes are defined. The average areal extent for each size class is calculated using the average volume, and the relationship described in Harbaugh (1977) and Power (1990), that is, area $= a \times \text{volume}^b$, where $a > 0$ and $0 < b < 1$. The average areal extent for each size class is raised to the power of discovery efficiency to reflect the magnification of the influence of areal extent on the probability with which a field is discovered. It is necessary to add a dryholes size class into the frequency-size distribution in order to reflect the exploration risk. The number of dryholes is calculated by subtracting the total area of all fields available (from sampled data) from the total area of the play or basin, and dividing by the area surrounding any individual well, based on the minimum distance between adjacent wells.

After obtaining each frequency-size distribution, we select the numbers of exploratory wells of interest (e.g., 5, 10, and 15, . . . ,) and calculate the means and standard deviations of the total discovery volume for all numbers of exploratory wells using Manly's approximation method. Then, we calculate the minimum, maximum, and the two shape parameters of the Beta distribution for each sampled number of exploratory wells. The procedure is repeated for all frequency-size distributions for each selected number of exploratory wells. Subsequently, all Beta distributions for each well number are combined into a single Beta distribution of total discovery volume, incorporating geological uncertainties.

In brief, the procedure of incorporating uncertainty in geological parameters and dryholes into frequency-size distributions is given in the following steps.

Step 1. Obtain the field-size distribution from experts (Fig. 1A).

Step 2. For each replication, sample from the number of fields distribution (Fig. 1B).

Step 3. With the number of fields obtained from Step 2, sample from the field-size distribution of Step 1.

Step 4. Categorize field data obtained from Step 3 into size classes.

Step 5. Calculate the average volume, average areal extent, and average areal extent raised to the power of discovery efficiency for each size class as described.

Step 6. Calculate the number of dryholes in dryholes size class as described. As a result of Steps 2–6, obtain one frequency-size distribution. From this distribution, continue with the following steps, based on the methodology described in Chungcharoen and Fuller (1996).

Step 7. Select the numbers of exploratory wells (e.g., 5, 10, 15, . . .) and calculate the expected values and standard deviations for all numbers of exploratory wells using Manly's approximation method.

Step 8. Calculate the maximum total discovery volumes for all exploratory wells. Note that the minimum total discovery volume for these exploratory wells is zero, which indicates that all exploratory wells are dry.

Step 10. Use the methods in Chungcharoen and Fuller (1996) to compute the two shape parameters of the Beta distributions. Generate the Beta distributions for selected exploratory wells. For each selected exploratory well, separate the Beta distribution into several total discovery intervals with the interval width of 500 b.c.f., for example, 0–500, 500–1000, ... Then integrate the Beta density function between these intervals to determine the areas under the density function which represent the probabilities that the total discovery volumes would fall into these ranges.

Steps 2–10 are repeated *n* times to create *n* Beta distributions for each selected number of exploratory wells. The probabilities in all intervals from *n* Beta distributions for each selected number of exploratory wells are averaged, and a single Beta distribution is fitted to the averages to obtain the distribution of total hydrocarbon discovery volume incorporating geological uncertainties. Chungcharoen (1997) discusses a time-saving approximation, in which the parameters of the *n* Beta distributions are averaged directly to produce the single Beta distribution.

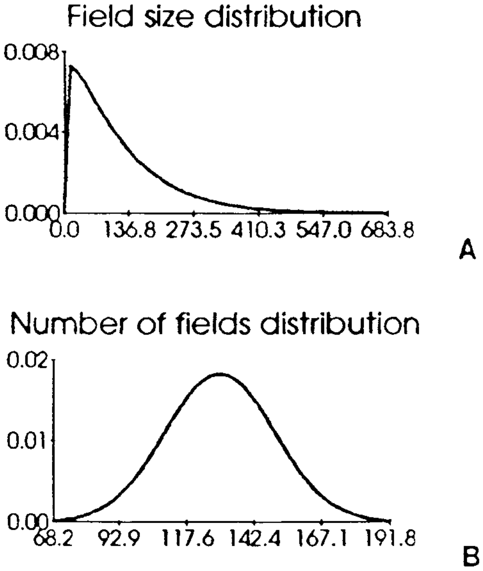

Figure 1. Examples of field-size distribution, A; and number of fields distribution, B.

Economic Evaluations

Next, bring economic parameters into the calculations by constructing a probability distribution of the net present value (NPV) of exploration, development, and production investments. As explained later, the basic methodology of the last section must be modified for this purpose. The major economic parameters are the oil/gas price, costs of exploration, development, production, and the discount rate. We exclude taxes and royalties from the calculations in this paper because the details differ from one jurisdiction to another; there is no difficulty, in principle, to include them. The real value of future cash flows is estimated and, therefore, we use a real discount rate. There is no risk adjustment in the discount rate; uncertainties about dryholes and discovery sizes will be reflected in the calculated distribution of NPV. Data for natural gas fields offshore of Nova Scotia are used for illustration in this paper.

For the price parameter, the netback gas price received by operators of the offshore Nova Scotia Shelf fields is used (see discussion regarding this price in Chungcharoen, 1997). Costs generally are classified as exploration, development, or production costs. Exploration costs normally are incurred on a yearly basis as basins are explored in proportion to the number of exploratory wells, whereas development and production costs are incurred only when there is a significant discovery and companies believe the potential revenue from developing a field will yield an acceptable profit. Based on several studies regarding the relationship between costs and influence factors, researchers have determined that costs of fields in a given region are related primarily to whether they are developed, and to field size (e.g., Nystad, 1981; Attanasi and others, 1981; Attanasi and Haynes, 1984; Power, 1990). It seems to be reasonably accurate to assume that such costs are the sum of a fixed cost (different for dryholes and discoveries) and costs that are proportional to field size measured as volume. Therefore, when aggregating costs over a region, costs depend on total volume discovered, on the number of exploratory wells, and on the number of discoveries.

Note that all costs regarding exploration and development wells can differ from one region to another depending on several influence factors. For example, exploration and development costs in an offshore region generally are higher than the same costs onshore.

We perform NPV evaluation because it is one of the basic tools used to evaluate and compare investment opportunities. Many exploratory wells will turn out to be dry as a result of the tremendous risk involved. NPV evaluations are performed to justify whether each discovered field will be viable to develop commercially. A field having an excess of discounted net revenue over discounted postexploration costs is declared economic; other fields are declared subeconomic and removed from the discoveries. When considering the overall exploration program, the discounted net revenue accruing to the company from the operation of these economic fields should exceed the discounted costs of developing and operating all economic fields plus the discounted costs of exploration in the basin in order to sustain the exploration program. Our method produces a probability distribution on this NPV of exploration.

Because there is no specific rule that directs the time rate of exploration activity for each company, we assume that the company has in mind an exploration program with a specific number of exploratory wells in each of several years of an exploration program, that is, IJ exploratory wells in I years, at the rate of J exploratory wells each year.

In addition to this consideration, there also is no obvious method of predicting the rate at which discovered fields will be developed. Therefore, we assume that the company begins developing the discovered field on a fixed schedule (e.g., within 3 months after a field is discovered). From these assumptions, the distribution of NPV for a particular exploration program can be determined.

To compute the NPV of total annual revenue resulting from J exploratory wells each year, we consider the revenues from selling the amount of oil/gas that has been discovered after production has begun. In general, the production of an oil/gas field declines with time. Because the exponential decline curve has been used most widely to represent the production decline characteristics of oil/gas fields (see for example, Newendrop, 1975; McCray, 1975; and Ikoku, 1985), we adopt the exponential decline curve for the purpose of demonstration of our methodology. Other forms of production decline curves could be assumed, if desired, for example, constant production for several years, dictated by pipeline capacity, followed by exponential decline.

The NPV for a particular exploration agreement can be written as:

$$\text{NPV of the project} = \sum_{i=1}^{I} \frac{(\text{NPV of total annual revenues} - \text{NPV of total costs})_i}{(1+r)^i} \tag{6}$$

where I is the total years of the exploration program, and r is a real discount rate. Each of the NPVs in the summation is taken to the year i. The company will drill J exploratory wells each year, so the total number of exploratory wells drilled by the 1st, 2nd, 3rd, . . . , Ith years will be, J, $2J$, $3J$, . . . , and IJ exploratory wells, respectively.

Each of the revenue terms in the summation is of the following form:

$$\text{NPV of total annual revenues} = \sum_{t=t_0+1}^{T+t_0} \frac{1}{(1+r)^t} (\text{Price})\, V_0 e^{-\delta t}, \tag{7}$$

where t is the number of periods after discovery, t_0 is the delay between discovery and startup, T is the total number of periods of production, V_0 is the volume produced during the first year (proportional to the total volume discovered during that year's exploration), and δ is the exponential decline rate.

Each of the cost terms in the project NPV expression is of the following form:

$$\begin{aligned} \text{NPV of total costs} &= \text{Total initial costs} \\ &\quad + \text{NPV total annual costs} \\ &= \{AJ + B(J - n_1) + CV\} \\ &\quad + \sum_{t=t_0+1}^{T+t_0} \frac{1}{(1+r)^t} \{D(J - n_1) \\ &\quad + EV + h_1 V_0 e^{-\delta t}\}, \end{aligned} \tag{8}$$

where A, B, C, D, E, and h_1 are constants for the area under exploration, J is the number of exploratory wells drilled, n_1 is the number of dryholes and subeconomic discoveries, and V is the total volume discovered during the year in question (see Chungcharoen, 1997, for details). With the assumption of exponential decline, V is proportional to V_0 in the usual way, as discussed by Chungcharoen (1997).

To determine the distribution of total costs (and of NPV), a distribution of the number of economic discoveries, $J - n_1$, is needed. In order to approximate quickly the distribution of the number of economic discoveries, some information from Manly's approximation in a binomial distribution is used. Because Manly's approximation gives the approximate mean number of fields remaining undiscovered in each size class after J exploratory wells, we are able to compute the approximate mean number of economic discoveries. Thus, we are able to calculate the probability of success for a binomial distribution by dividing this mean value by the number, J, of exploratory wells.

To verify the use of the binomial distribution to represent the distribution of the number of economic discoveries, we compared the binomial distribution to the distribution of the number of discoveries resulting from the exact simulation of the probabilistic model of hydrocarbon exploration process by using several frequency-size distributions as input data. The comparison between the two distributions is done by histograms and the chi-square (χ^2) tests. For the data tested, the binomial is a good fit (see Chungcharoen, 1997, for details).

Upon knowing the distributions of number of discoveries and total volume for J, $2J$, $3J$, . . . , IJ exploratory wells, a Monte Carlo approach is used to generate distributions of total costs and revenues for each number of J exploratory wells. To account for dependencies between distributions of the number of discoveries and total volume, and between distributions for different years, a conditional sampling process must be used as now outlined. We describe the conditional sampling for the example $I = 3$ years; the extension to $I > 3$ should be clear.

Step 1. From the input of n frequency-size distributions, obtain the average probabilities of the total number of economic discoveries and the success probabilities of the binomial distributions for J, $2J$, $3J$ exploratory wells.

Step 2. Obtain the average values of the mean, standard deviation, minimum value, maximum value, and the two shape parameters of distributions of total hydrocarbon discoveries (approximate Beta distributions) for 1, 2, . . . , $3J$ discovery wells, following the methods of incorporating uncertainty in geological parameters in the last section. Note that the resulting Beta distributions do not include the effects of dryholes: the distribution that is required in later steps is for the total volume at a given *discovery* number.

First Year Exploration

Step 3. Sample from the binomial distribution using the probability parameter from Step 1 to obtain the total number of economic discoveries, k_1 from J exploratory wells.

Step 4. With this sampled value from Step 3, sample from the Beta distribution for $k_1 < J$ discoveries as given in Step 2. After completing this step, we obtain a sample of the number of economic discoveries and the corresponding total volume from J exploratory wells in the first year.

Second Year Exploration

Step 5. For the second year of exploration with another J exploratory wells, sample from the binomial distribution representing the distribution of total number of economic discoveries based on a total of $2J$ exploratory wells using the probability of success from Step 1 in order to obtain the number of economic discoveries, k_2. This value, however, gives the number of discoveries from a total of $2J$ exploratory wells independently from the first year exploration. In order to be more realistic, two constraints must be put into the sampling process to obtain the conditional sample of total number of economic discoveries from the total of $2J$ exploration wells. First, the sample of the total number of economic discoveries cannot be less than the total number of economic discoveries in the first year, that is, $k_2 \geq k_1$. Second, the sample cannot be greater than the total number of economic discoveries in the first year plus the maximum number of J discoveries in the second year, that is, $k_2 \leq k_1 + J$. If either constraint is violated, then the sampling is repeated until the constraints are satisfied.

Step 6. With the sample k_2 of total number of economic discoveries, sample from the Beta distribution for $k_2 < 2J$ discoveries, with parameters given in Step 2, to obtain the corresponding total volume. In this step, one more constraint must be added to the sampling process: the total volume obtained after two years must not be less than the total volume obtained from the first year. If this constraint is violated, Step 5 and this step must be repeated. After completing this step, we obtain the total number of discoveries and the corresponding total volume after 2 years of $2J$ exploration wells.

Step 7. In order to obtain the number of economic discoveries and the corresponding total volume obtained, within the second year, subtract both the total number of economic discoveries and the corresponding total volume of the first year from the total number of discoveries and the total volume after 2 years.

Third Year Exploration

Step 8. Sample k_3 from the binomial distribution using the probability of successful discoveries obtained from Step 1 for a total of $3J$ exploratory wells. Again, two constraints must be imposed into the sampling process. $k_2 \leq k_3 \leq k_2 + J$. If either constraint is violated, this step must be repeated.

Step 9. Sample from the Beta distribution for $k_3 < 3J$ discoveries, from Step 2 to obtain the corresponding total volume. Again, the total volume obtained after 3 years must not be less than the total volume obtained from the 2 years of exploration. If this constraint is violated, Step 8 and this step must be repeated. After completing this step, we obtain the total number of discoveries and the corresponding total volume after 3 years of $3J$ exploratory wells.

Step 10. In order to obtain the number of economic discoveries and the total volume obtained, within the third year, subtract both the total number of discoveries and the corresponding total volume after 2 years from the total number of discoveries and the corresponding total volume obtained in 3 years.

Step 11. After obtaining the number of economic discoveries and the total volume of discoveries in the first year, the conditional total number of discoveries and the conditional total volume discoveries in the second year, and the conditional total number of discoveries and the conditional total volume discoveries in the third year, calculate the NPV as given by expressions (1)–(3).

Step 12. Repeat Steps 3–11 for 10,000 replications to obtain the distribution of number of discoveries, the corresponding distribution of total volume, the distribution of NPV of total annual revenues, the distribution of NPV of total costs, the distribution of net profit for every year, and, finally, the distribution of NPV for a 3-year exploration program.

The given procedures have been coded in FORTRAN and the program runs on a PC. The code is suitable for our research, but is not sufficiently well documented or user-friendly for use by others.

RESULTS

We illustrate our methodology with data for the offshore Nova Scotia Shelf which is partly explored and yet considered to be a frontier basin. Following Power (1990), we use the Weibull distribution, to represent the field-size distribution. According to Power (1990), the shape (α) and scale (β) parameters of the fitted Weibull distribution to the Nova Scotia Shelf data were estimated to be 0.869 and 117.68, respectively. In order to represent the uncertainty of the number of fields, we assume a triangular distribution with the minimum, most likely, and the maximum values of 100, 113, and 130, respectively. The form of this distribution is not a key part of the methodology; the analyst may use any distribution that properly represents this uncertainty. The total productive area of the Nova Scotia Shelf was averaged using information from a COGLA map and a Jansa and Wade map to be 30,132 square miles, which covers an area around Sable Island that has been the center of exploration drilling (Power, 1990). We assume a 3-year exploration program for which the company has a fixed drilling schedule of five exploratory wells each year. We examine several situations to illustrate the benefits of using our methodology (see details in Chungcharoen, 1997).

First, we compare the results of our methodology to the results with only a single frequency-size distribution, which presents one possibility of frequency-size distributions that might occur in the Nova Scotia Shelf. This single frequency-size distribution, which is generated by sampling only once from the number of fields distribution, does not take into account the uncertainty in geological parameters. Figure 2 shows the comparison of the distributions of the total discovery volume from our methodology and from a single frequency-size distribution for five exploratory wells.

As a single frequency-size distribution presents only one possibility of a frequency-size distribution that might occur in the Nova Scotia Shelf because of uncertainties in geological parameters, the distribution of total discovery volume resulting from incorporating the uncertainties by averaging all Beta distributions in our methodology will be a better representation because it reflects the reality about uncertainties involved in exploration in the basin. If the uncertainties involved are not large, the differences between these two distributions are small, for example, when a company has experience in the basin. Therefore, in a well-established basin, a single frequency-size distribution might be sufficient to be used in the model. However, when uncertainties become large in a frontier basin where geological information is limited, the distributions used to represent the number of fields and field-size distribution of that basin will have wider ranges. As a result, the differences between the distributions of total discovery volume between these two situations will become prominent and there will be some benefit of including uncertainties in geological parameters.

Next we illustrate the effect of increasing uncertainties of geological parameters on the distributions of total discovery volume. We doubled the standard deviation of the field-size distribution and the number of fields distribution one at a time while keeping both means unchanged. We determined that an increase in the uncertainty in field size alone causes a substantial increase in expected total discovery volume. Figure 3 shows the comparison of the distributions of the total discovery volume for five exploratory wells when the standard deviation of field size distribution is doubled.

We can see from Figure 3 that when the standard deviation is doubled, the distribution of the total discovery volume is more spread over wider ranges as a result of more uncertain information. The expected total discovery volume for five exploratory wells in the situation of regular standard deviation is 0.6202 t.c.f., whereas the expected total discovery volume of double standard deviation is 1.4087 t.c.f. Although this result seems to be counterintuitive to those unfamiliar with exploration models, it can be explained. Because the field-size distribution is highly right-skewed, as we double its standard deviation to increase the uncertainty and keep its mean unchanged; its density will spread toward the right tail. Hence, the probability associated with large field sizes on the right tail is increased. By using the probabilistic model of the discovery process, the probability of discovering a field of one particular size class is proportional to the number of fields of that size class and its area raised to the power of discovery efficiency. As a result, with double standard deviation, there will be a higher probability that large fields will be discovered at an early exploration stage. Figure 4 shows the comparisons of the distributions of NPV of the exploration program when the standard deviation of the field-size distribution is doubled.

From Figure 4, the distribution of NPV for double uncertainty spreads toward the right-hand side of the graph more than the distribution of NPV for regular uncertainty. The expected net present values of the exploration program in both situations are \$1.5586 and \$5.2517 billion. We can see that the expected net present value for double uncertainty is approximately 3.4 times the expected net present value of regular

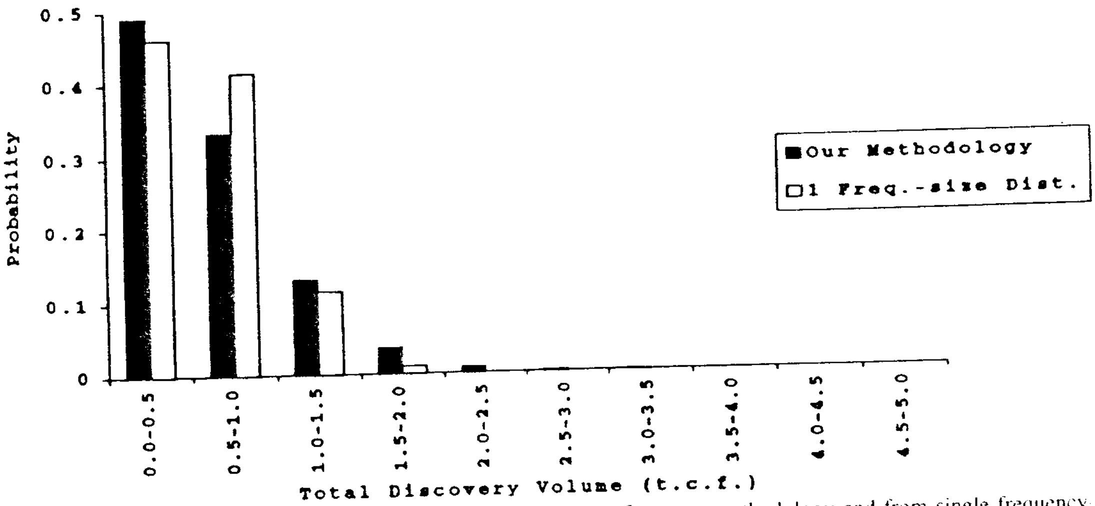

Figure 2. Comparison of distributions of total discovery volume from our methodology and from single frequency size distribution for five exploratory wells.

uncertainty. This is because the distribution of total volume with double uncertainty is spread more toward the right-hand side of the graph as explained. This raises an intriguing question: might decision-makers actually prefer to explore in a region whose field-size distribution shows greater uncertainty? As discussed by Chungcharoen (1997), the answer depends on one's interpretation of a larger standard deviation in the field-size distribution—greater ignorance about field sizes, or different real geological conditions, which produce a larger spread in field sizes (including a greater probability of large fields).

We continue our illustration of the use of the methodology by showing the effects of price changes on the NPV distribution. Figure 5 shows the distributions of NPV with the prices of \$3.08, \$3.80, and \$4.58/m.c.f. at a discount rate of 4.25%. This rate may seem low, but keep in mind that it is both real, that is, inflation-free, and risk-free, because the risk is represented in the distribution of NPV. The prices used here are the netback prices (in 1995 dollars), which cover the worst, the reference, and the best expectation of prices perceived in the long-run by offshore Nova Scotia producers (Chungcharoen, 1997).

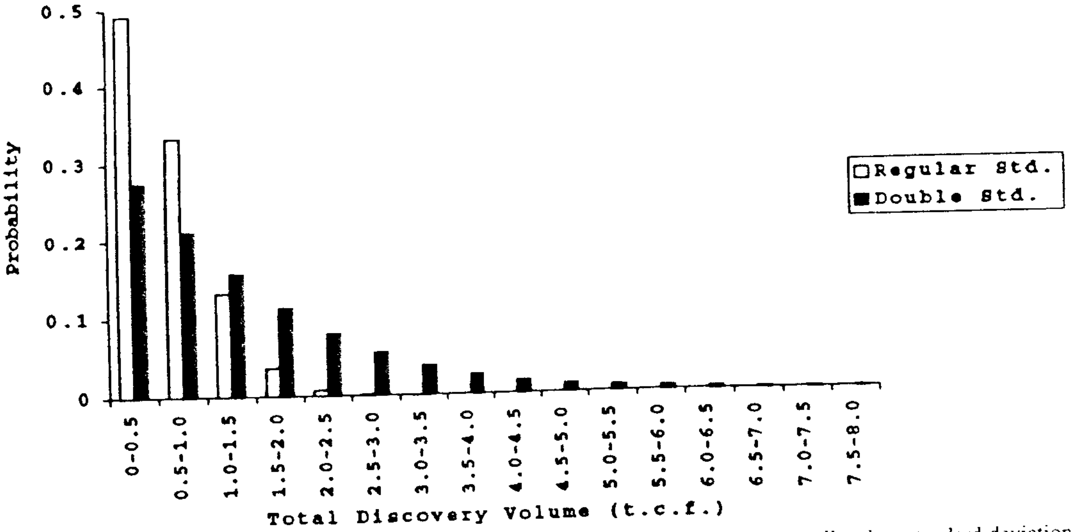

Figure 3. Comparison of distributions of total discovery volume for five exploratory wells when standard deviation of field-size distribution is doubled.

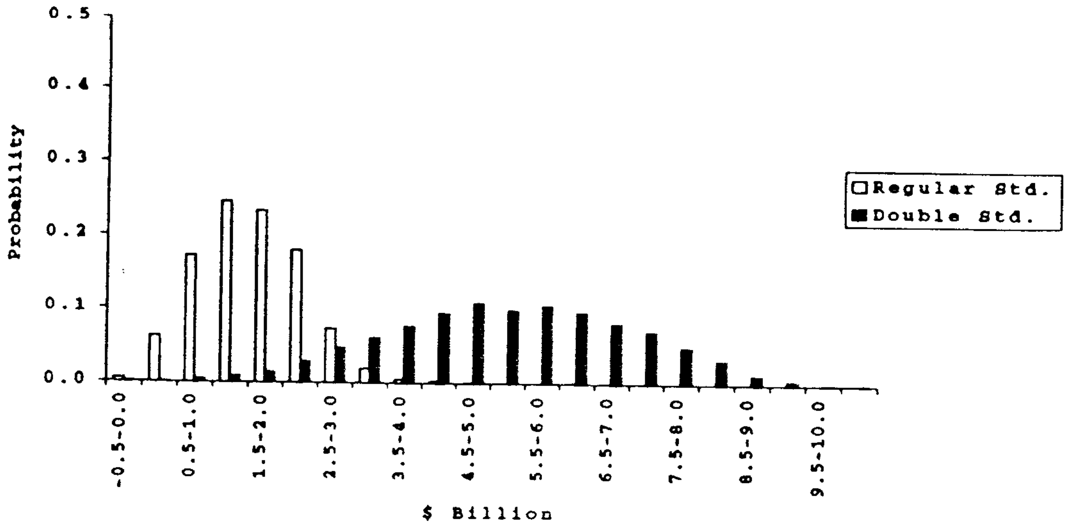

Figure 4. Comparison of distributions of NPV for exploration program when standard deviation of field-size distribution is doubled with reference price of $3.80/m.c.f. and real discount rate of 4.25%.

From Figure 5, we can see clearly that reducing the price from $3.80 to $3.08/m.c.f. causes the distribution of NPV to move toward the left-hand side of the graph and to have a narrower range. This results in the reduction of expected net present value of the exploration program. On the other hand, raising the price from $3.80 to $4.58/m.c.f. causes the distribution of NPV to spread more toward the right-hand side resulting in an increasing expected net present value of the program. Notice that there is only slight movement at the left tail of the distribution of NPV as price changes. These results can be explained by considering the effect of changes in price on the minimum economic field-size class. Dropping the price from $3.80 to $3.08/m.c.f. will cause fields in the minimum economic size class to become subeconomic, and, hence, are not developed. This will affect the distribution of the number of economic discoveries and, subsequently, the distribution of total discovery volume in each frequency-size distribution. As these distributions are used in the economic evaluations, the revenues generated from fields in the minimum economic size class will be absent. In addition, the revenues from already commercially declared fields are reduced because of

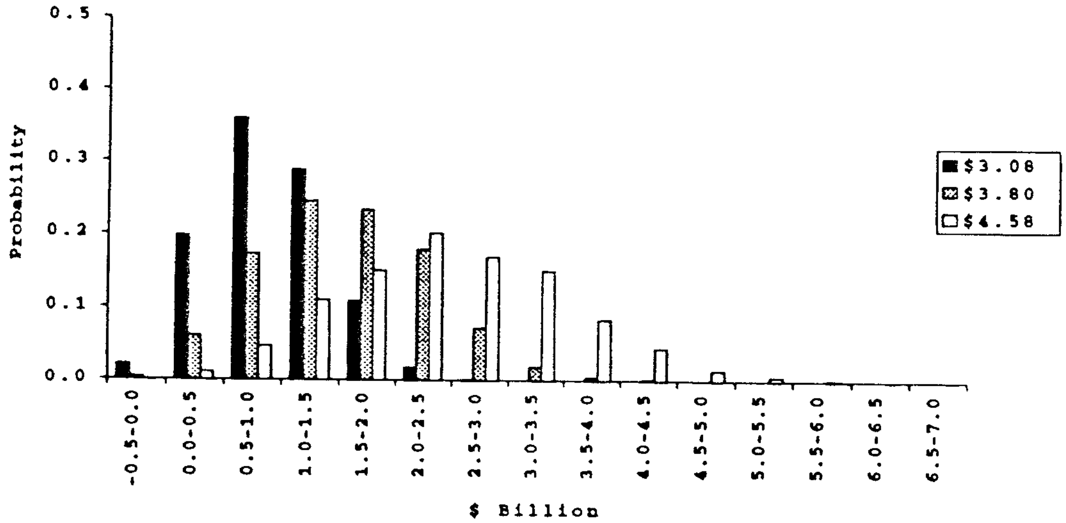

Figure 5. Distributions of NPV with price of $3.08, $3.80, and $4.58/m.c.f. at discount rate of 4.25%.

the reduction in operation life of these fields. On the other hand, raising the price from \$3.80 to \$4.58/m.c.f. has no effect on the minimum economic field-size class. However, raising the price will increase the revenues from additional production resulting from the extensions to the economic life of already commercially viable fields.

CONCLUSIONS

A major benefit of our methodology is that it allows the model to run faster than the regular simulation method of the probabilistic model of hydrocarbon exploration. This allows the model to incorporate uncertainty in geological parameters into the frequency-size distribution data, yet obtain the resulting distributions within a short period of time: 12 min. for each scenario of the results, versus 16 h with regular simulation, on a desktop Pentium PC. Within this short period of time, the model allows a company to analyze thoroughly a variety of exploration and development scenarios with changes in geological and economic conditions. This methodology is valuable especially because of the paucity of work related to frontier exploration. This methodology could be used by a company as a part of a planning system for exploration programs. In addition, results from this methodology could assist government departments by supporting their efforts to establish the potential of hydrocarbons discoveries, to predict future exploration activity, and to aid in their analyses of policies concerning exploration programs regarding taxes and royalty regimes in any basin with various stages of exploration activity.

The main limitation of this methodology however, is in the accuracy of the approximation of using a binomial distribution to represent the distribution of the number of discoveries. The accuracy of the approximation deteriorates as the number of exploratory wells increases.

DIRECTIONS FOR FUTURE RESEARCH

Several stages of exploration in the basin should be investigated further to demonstrate completely the validity of this methodology throughout the exploration process. A more formal model evaluation including comparing model predictions with a few real data sets should be considered. This methodology should be applied to other frontier basins in order to assess the usefulness of Manly's approximation method and the claim of using a family of Beta distributions to represent the approximate distribution of the total discovery volume as well as using a binomial distribution to represent the approximate distribution of the number of discoveries. As explained in Chungcharoen (1994), there are small errors of the means and the standard deviations between the simulation method and Manly's approximation method. These errors are carried into the later calculations. Therefore, a systematic approach to propagation of errors in the approximation method should be thoroughly investigated. To reduce these errors, a regression model, as suggested by Ninpong (1992), might be used to adjust the approximated means and standard deviations in order that the Beta distributions and the binomial distributions can provide a better fit to the distributions of the total discovery volume and the distributions of the number of discoveries, respectively. In addition, the accuracy of the approximation could be improved by modifying parameters of the probability mass function of the binomial distribution as suggested by Johnson, Kotz, and Kemp (1992).

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