A Hybrid Option Pricing Model Using a Neural Network for Forecasting Volatility

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Abstract

- The Black-Scholes model is the standard approach used for pricing financial options.
- Option prices valued by the model often differ from the prices observed in the financial markets.
- This research applies a hybrid neural network for improving the estimation of option market prices.
Fundamentals

Options

- An option is the right, but not the obligation to take action in the future
  - Call option: Right to buy the asset at the strike price
  - Put option: Right to sell the asset at the strike price

- In-the-Money: Exercise price produces profit
- At-the-Money: Exercise price equals primitive price
- Out-of-the-Money: Exercise price is unprofitable
Black-Scholes Model

- The Black-Scholes Option Pricing Model
  - Ideal conditions
    - Assumes lognormal distribution of prices
    - Assumes constant volatility
- Variables
  - Known Variables
    - Stock Price ($S_0$)
    - Strike Price ($X$)
    - Risk-free interest rate ($r$)
    - Time to expiry ($T$)
  - Unknown Variable
    - Volatility ($\sigma$)
  - Output
    - Option price

\[
C = S_0 N(d_1) - X e^{-rT} N(d_2)
\]

\[
P = X e^{-rT} N(-d_2) - S_0 N(-d_1)
\]

\[
d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma \sqrt{T}}
\]

\[
d_2 = \frac{\ln(S_0 / X) + (r - \sigma^2 / 2)T}{\sigma \sqrt{T}}
\]
Volatility

- **Historical Volatility**
  - Standard Deviation of a stock’s return over a fixed period of time

  \[ HV = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2} \]

  where \( X_i \): Return at end of \( i^{th} \) interval

- **Implied Volatility**
  - Derived from an option pricing model by adding all known variables and market option price into the formula
Volatility Smile

- The relationship between the implied volatility of an option as a function of its strike price is known as a volatility smile.

<table>
<thead>
<tr>
<th>Implied Volatility</th>
<th>Strike Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) FX Option</td>
<td></td>
</tr>
<tr>
<td>b) Equity Option</td>
<td></td>
</tr>
</tbody>
</table>
Neural Networks

- Non-linear problems
  - Stock price forecasting
  - Mutual fund performance forecasting
- No rules/assumptions are needed
- Can learn from past data
Option Model: 3 models

<table>
<thead>
<tr>
<th>No.</th>
<th>Volatility Estimation</th>
<th>Option Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. HV-BS</td>
<td>Historical Volatility</td>
<td>Black-Scholes</td>
</tr>
<tr>
<td>2. NN-BS</td>
<td>Neural Network</td>
<td>Black-Scholes</td>
</tr>
<tr>
<td>3. NN-HB</td>
<td>Neural Network</td>
<td>Hybrid</td>
</tr>
</tbody>
</table>
Option Pricing Model

1. Historical volatility: Black-Scholes model (HV-BS)
   Comprised of 2 parts
   1. Historical Volatility : Estimate volatility
   2. Black-Scholes Model : Estimate the option prices

Fig. 1 : HV-BS option pricing model
2. Neural Network : Black Scholes (NN-BS)

Comprised of 2 parts

1. Neural Network : Estimate volatility
   \[ \text{Volatility} = f(s_0, x, T, r) \]

2. Black-Scholes Model : Estimate the option prices

Fig. 2 : NN-BS option pricing model
Option Pricing Model

3. Neural network to estimate volatility with a hybrid model (NN-HB)

Comprised of 2 parts

1. Neural Network: Estimate volatility
   \[ \text{Volatility} = f(s_0, x, T, r) \]

2. Hybrid Network: Estimate the difference between the Black-Scholes model result and the actual market option prices
3. Neural network to estimate volatility with a hybrid model (NN-HB) (cont.)

- Option price is a summation between the Black-Scholes model and the network output.

Fig. 4: NN-HB option pricing model
Data Set

- Five different underlying stocks
- Focused on the closing prices
- Training data set (3 months)
  - July 1, 2002 - September 30, 2002
- Testing data set (15 days)
  - October 1, 2002 - October 15, 2002
- Measure performance
  - Mean squared error (MSE)
  - Mean absolute error (MAE)
Data Set (cont.)

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<thead>
<tr>
<th>Underlying Stock</th>
<th>Number of data records</th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>All (in and out)</td>
<td>Train</td>
<td>Test</td>
<td>Train</td>
<td>Test</td>
<td>Train</td>
<td>Test</td>
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<td>1850</td>
<td>304</td>
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<tr>
<td>MCD</td>
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<tr>
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<tr>
<td>C</td>
<td></td>
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<td>2399</td>
<td>292</td>
<td>4053</td>
<td>826</td>
</tr>
</tbody>
</table>

Table 1. Amount of training and testing data for each stock
## Results

<table>
<thead>
<tr>
<th>Stock</th>
<th>All (in and out)</th>
<th>In-the-money</th>
<th>Out-of-the-money</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HV-BS</td>
<td>NN-BS</td>
<td>NN-HB</td>
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<tr>
<td>KO</td>
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<td>C</td>
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Table 2. Mean squared error results
## Results (cont.)

<table>
<thead>
<tr>
<th>Stock</th>
<th>All (in and out)</th>
<th>In-the-money</th>
<th>Out-of-the-money</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HV-BS</td>
<td>NN-BS</td>
<td>NN-HB</td>
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<tr>
<td>KO</td>
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<tr>
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<tr>
<td>C</td>
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<tr>
<td>IBM</td>
<td>0.7146</td>
<td>0.1451</td>
<td>0.0844</td>
</tr>
</tbody>
</table>

**Table 3. Mean absolute error results**
Conclusions

- The hybrid neural network is shown to improve the performance of the Black-Scholes model to capture deviations from the strict assumptions.
- When combining a neural network for estimating volatility with the hybrid neural network, superior performance is achieved.