Value-at-Risk and Expected Shortfall under Extreme Value Theory Framework: An Empirical Study on Asian Markets

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Abstract

We propose methods in estimating Value-at-Risk (VaR) and expected shortfall (ES); the conditional loss over VaR. Our methodology incorporates the two popular conditional volatility models namely GARCH and exponential weighted moving average (EWMA) in estimating current volatility and applying extreme value theory (EVT) in estimating tail distribution. This study covers ten Asian equity markets, which are Hong Kong, Japan, Singapore, China, Indonesia, Korea, Malaysia, Philippines, Taiwan and Thailand during the period 1993 to 2007. As expected, our conditional EVT models outperform other models with normality assumption in almost all the cases. On the other hand, the conditional EVT models are not trivially different from the filtered historical simulation model (or conditional HS). Regarding the conditional volatility models, even though GARCH can reflect more flexible adjustment than EWMA, the result shows that the simpler EWMA-based models are still useful in both VaR and ES estimation.

Keywords: Risk Management; Value-at-Risk (VaR); Expected Shortfall (ES); Fat-tailed; Extreme Value Theory (EVT); Volatility Prediction; GARCH; Exponential Weighted Moving Average (EWMA); Tail Estimation

JEL Classifications: C52, G10
1. Introduction

In financial industry today, Value-at-Risk (VaR) is the most popular risk measure widely used by financial institutions to measure and ultimately to manage the market risk which represents the primary concern for regulators and internal risk controllers. VaR refers to the maximum potential significant loss over a given period at a certain confidence level. However, VaR is often criticized and suffers from inconsistencies because of not being a coherent risk measure as proved in Artzner et al. (1999) and Acerbi and Tasche (2002). Furthermore, it ignores statistical properties of the significant loss beyond VaR or tail risk. In order to overcome those drawbacks, the expected shortfall (ES) is conducted as an alternative risk measure.

Expected shortfalls or the tail conditional expectation was first proposed by Artzner et. al.(1997). It is defined as the conditional expectation of loss for losses beyond VaR level. By definition, ES takes account of tail risk and also demonstrates sub-additivity property which assures its coherence as a risk measure. The more extension of its plausible coherence can be found in Inui and Kijima (2005). We refer to Yamai and Yoshiba (2002a, 2002b, 2002c, 2002d, 2005) for further studies on expected shortfall and its application.

Most of financial concepts developed in the past decades rest upon the assumption that returns follow a normal distribution. This is the most well-known classical parametric approach in estimating VaR and ES namely the Gaussian approach. However, empirical results from McNeil (1997), Da Silva and Mendez (2003) and Jondeau and Rockinger (2003), show that extreme events do not follow Gaussian paradigm. Obviously, the empirical returns, especially in the high frequency fashion are characterized by heavier tails than a normal distribution.

To avoid ad-hoc assumption, the historical simulation (HS) approach is often used as an alternative. This non-parametric approach is conceptually simple to implement and has several advantages. It employs recent historical data so it allows the presence of heavy tails without making assumption about the probability distribution of the losses, therefore, no model risk. Even though HS is potentially accurate, it suffers from some serious drawbacks. First, an extrapolation beyond past observations tends to be inefficient with high variance estimator especially for small observations. Moreover, to mitigate the above problems by considering longer sample periods, the method tends to face the time-varying of return volatility.

Beyond traditional approaches, various alternative distributions have been proposed to describe fat-tail characteristics. One of the most popularity is based on Extreme Value Theory (EVT). This method focuses on the tails behavior of distribution of returns. Instead of forcing a single distribution for the entire sample, it investigates only the tails of the return distributions, given that only tails are important for extreme values. Many have viewed the EVT in finance such as Embrechts et al (1999), Bensalah (2000), Bradley and Taqqu (2002) and Brodin and Klüppelberg (2006). To investigate the extreme events, McNeil (1997) apply method using EVT for modeling extreme historical Danish major fire insurances losses. The result indicates the usefulness of EVT in estimating tail distribution of losses. Not only is the EVT approach a convenient framework for the separate treatment of the tails of a distribution, it allows asymmetry as evidence in LeBaron and Samanta (2005).

Presently, the Basel II has been imposed to financial institution to meet the capital requirement based on VaR. Hence, the effectiveness of methodologies has become more concerned. Gençay and Selçuk (2004) have reviewed VaR estimation in some emerging markets using various models including EVT. The empirical result shows that EVT-based model provides more accurate VaR especially in a higher quantile. In depth, the generalized Pareto distribution (GPD) model fits well with the tail of the return distribution. Harmantzis
et. al. (2005) and Marinelli et. al. (2006) have presented how EVT performs in VaR and ES estimation compared to the Gaussian and historical simulation models together with the other heavy-tailed approach, the Stable Paretian model. Their results support that fat-tailed models can predict risk more accurately than non-fat-tailed ones and there exists the benefit of EVT framework especially method using GPD. Furthermore, Gilli and Këllezi (2006) try to illustrate EVT using both block maxima method (BMM) and peak over the threshold (POT) in modeling tail-related risk measures; VaR, ES and return level. They find that EVT is useful in assessing the size of extreme events. In depth, POT is proved superior as it better exploits the information in sampling.

The stochastic volatility structure of financial return time series is also important issue because the presence of stochastic volatility implies that returns are not necessarily independent over time. An extreme value in high volatility period appears less extreme than that in the period of low volatility. Consequently, it is reasonable to consider the two stylized facts, stochastic volatility and the fat-tailedness of conditional return distribution. McNeil (1999) proposes some theoretically point of view on EVT framework as tool kits of risk management and emphasizes the generality of this approach via stochastic volatility. Also, McNeil and Frey (2000) have focused on the stochastic volatility and fat-tailedness of conditional return distribution over short time horizons. They propose two-step procedure for VaR and ES estimation by combining GARCH-based modeling to estimate current volatility and using EVT framework for estimating tail of the distribution. The empirical result reveals that their method gives better one-day estimates than method which ignores heavy-tail. The similar result in VaR estimation is also found in Fernandez (2005). Nyström and Skoglund (2002) also test on VaR and ES by applying ARMA-GARCH with EVT framework to estimate extreme quantiles of univariate portfolio risk factors. Comparing to those of RiskMetrics, conditional t-distribution and empirical distribution, the study shows that refined methods like the EVT give a significant contribution in estimation risk.

Various literatures employ ARCH/GARCH family to overcome the drawback emerged from heteroscedasticity and/or to filter the volatility clustering. On the other hand, the stochastic volatility model like exponential weighted moving average (EWMA) is also used owing to its simplicity. Christoffersen (2006) have theoretically suggested various models in VaR and ES estimation including dynamic volatility paradigm. In addition, Caporin (2003) conducts the study on the tradeoff between complexity and efficiency of VaR through Monte Carlo study using EWMA and GARCH as volatility estimation models. The result shows that EWMA provides a good approximation for daily VaR.

In this paper, we present how EVT framework performs under the select risk measures; VaR and ES in ten East Asian MSCI Country Indices. Following McNeil and Frey (2000), we employ two-step procedure to obtain estimates of the conditional VaR and ES. Rather than using only GARCH modeling in volatility prediction we contribute to the EWMA which is conventionally implemented in the RiskMetrics VaR. These two conditional volatility estimations will be deployed to demonstrate the stochastic volatility fashion, using EVT framework to generate the appropriated tail distribution of index losses. Our approach is compared to the Gaussian-based model and the historical simulation (HS). Furthermore, we extend our comparison with the filtered historical simulation (FHS) technique, proposed by Christoffersen (2006) in consideration of time-varying volatility. Obtaining selected approaches, we deploy the rolling window basis performed in Gençay and Selçuk (2004) and Harmantzis et al (2005) to estimate the relevant parameters as it incorporates the changes in market condition. Lastly, we evaluate the predictive accuracy of various models using backtesting procedure.

The benefits of this study are twofolds. First, it provides the better understanding and implication about VaR and ES in fat-tailed environment specifically in East Asian equity
markets. Second, the paper considers the tradeoff between the complexity and efficiency in risk measurement both in volatility prediction and the tail distribution of the financial returns.

The paper is organized as follows. Section 2 describes the theoretical background of each approach. Section 3 focuses on the methodology for computation VaR and ES. Section 4 presents the data employed and the preliminary data analysis. The empirical study and analysis of testing results will be in Section 5. Finally, Section 6 is conclusion.

2. Theoretical Framework

This section introduces the definitions of two risk measures namely, Value-at-Risk and expected shortfall and outlines the key concepts of theoretical framework used in the study which are Gaussian, historical simulation, and extreme value theory.

2.1. Definitions and Properties

2.1.1. Value-at-Risk (VaR)

Value at Risk is generally defined as possible maximum loss over a given holding period within a fixed confidence level. Mathematically, McNeil et al. (2005) define VaR, in absolute value, at $\alpha \in (0,1)$ confidence level ($\text{VaR}_\alpha(X)$) as follows.

$$\text{VaR}_\alpha(X) = \inf \{ x \mid P[ X > x ] \leq 1 - \alpha \} = \inf \{ x \mid F_X(x) \geq \alpha \},$$ (1)

where $X$ is the loss of a given market index, and $\inf \{ x \mid P[ X > x ] \leq 1 - \alpha \}$ indicates the smallest number $x$ such that the probability that the loss $X$ exceed $x$ is no larger than $(1-\alpha)$. Generally, VaR is simply a quantile of the probability distribution.

2.1.2. Expected Shortfalls (ES)

Expected shortfall (ES) is closely related to VaR. It is known as the conditional expectation of loss given that the loss is beyond the VaR level\(^1\).

$$\text{ES}_\alpha(X) = E \left[ X \mid X \geq \text{VaR}_\alpha(X) \right]$$ (2)

2.2. Theoretical Background

2.2.1. Gaussian Approach

The most famous parametric approach for calculating VaR and ES is based on Gaussian assumption. This is the simplest approach among various models to calculate VaR and ES. It is assumed the independent identical distribution of standardized residual terms. Under the assumption that loss $X$ is normal distributed with mean zero and variance $\sigma^2$, from equation (1) obtaining VaR,

$$\text{VaR}_\alpha(X) = \sigma \Phi^{-1}(\alpha)$$ (3)

where $\Phi$ denotes standard normal distribution function that $\Phi^{-1}(\alpha)$ is the $\alpha$-quantile of $\Phi$. Let $\phi$ be the density of the standard normal distribution, the corresponding $\text{ES}_\alpha(X)$ is,

\(^1\) An intuitive expression can be derived to show that ES can be interpreted as the expected loss that is incurred when VaR is exceeded. For mathematically prove, see McNeil et al. (2005) p.45.
\[ ES_\alpha (X) = \frac{\phi \left[ F^{-1} (\alpha) \right]}{1 - \alpha} \]  

(4)

### 2.2.2. Historical Simulation Approach (HS)

This approach is quite straightforward and intuitive. Firstly, we take a set of historical data, and use the observed losses to construct histogram of the underlying distribution. Then, losses are ranked from high to low values which allow determining the percentiles.

Let \( F_n \) denotes the empirical process of the observed losses \( X_i, i = 1, 2, 3, ..., n \).

\[ F_n (I) = \frac{1}{n} \sum_{i=1}^n I(X_i \leq I) \]  

(5)

where \( I(A) \) is the indicator function of event A, and \( X_i \) is identical independent distributed (i.i.d.) with unknown distribution \( F \). We obtain VaR using the empirical quantile estimation,

\[ VaR_u (X) = F_n^{-1} (\alpha) = X_{n(n)} \alpha, \quad \alpha \in \left( \frac{i-1}{n}, \frac{i}{n} \right) \]  

(6)

where \( X_{n(1)} \leq X_{n(2)} \leq \ldots \leq X_{n(n)} \) are the order statistics. Historical ES is as follows;

\[ ES_\alpha (X) = E [X | X > VaR_u (X)] = \left\{ \sum_{i=[na]} X_{n(i)} \right\} / (n - [na]) \]  

(7)

### 2.2.3. Extreme Value Theory (EVT)

Suppose the following \( X_1, \ldots, X_n \) be \( n \) observations and are all i.i.d sequences of losses with distribution function \( F_X (x) = P[X \leq x] \) and the corresponding \( Y_1, \ldots, Y_n \) are the excess over the threshold \( u \). We are interested in understanding the distribution function \( F \) particularly on its lower tail. Firstly, we describe the distribution over a certain threshold \( u \) using the generalized Pareto distribution (GPD) which is the main distributional model for excess over the threshold. The excess over threshold occurs when \( X_i > u \). Let denotes the distribution of the excess over the threshold \( u \) as \( F_u \) which is called the conditional excess distribution function,

\[ F_u (y) = P(X-u \leq y | X > u) = \frac{F(x)-F(u)}{1-F(u)}, \quad 0 \leq y \leq x_F - u \]  

(8)

where \( y = x-u \) for \( X > u \) is the excess over threshold and \( x_F \leq \infty \) is the right endpoint of \( F \).

**Theorem 1:** (Pickands (1975), Balkema and de Haan(1974)). For a large class of underlying distribution functions \( F \) the conditional excess distribution function \( F_u (y) \), for large \( u \), is well approximated by;

\[ \lim_{u \to \infty} \quad \text{sub}_{0 < \xi \leq \xi_u} \left| F_u (y) - G_{\xi, \beta(u)} (y) \right| = 0, \]

In the sense of the above theorem, our model of \( X_i \) with distribution of \( F \) assumes that for a certain \( u \) the excess distribution above the threshold may be taken to be exactly GPD for some parameters \( \beta \) and \( \xi \).
\[ F_u(y) \approx G_{\xi, \beta}(y) \]

where
\[
G_{\xi, \beta}(y) = \begin{cases} 
1 - \left( 1 + \frac{\xi y}{\beta} \right)^{-\frac{1}{\xi}}, & \xi \neq 0, \quad y \in \left(0, \frac{x_{\text{e}} - u}{\beta} \right], \quad \text{if } \xi \geq 0 \\
1 - \exp \left(-\frac{y}{\beta} \right), & \xi = 0, \quad \text{if } \xi < 0 
\end{cases}
\]

(9)

An additional parameter \( \beta > 0 \) has been introduced as scale parameter and \( \xi \) is the shape parameter or tail index. Both \( \beta \) and \( \xi \) are estimated via real data of excess returns. The tail index \( \xi \) gives an indication of the heaviness of the tail, the larger \( \xi \), the heavier the tail.

Assuming a GPD function for the tail distribution, analytical expressions for VaR and ES can be defined as a function of GPD parameters. Regarding to equation (9), we replace \( F_u \) with \( G_{\xi, \beta}(y) \) and \( F(u)^2 \) with the estimate \((n-N_u)/n\). Denotes that \( n \) is total number of observations and \( N_u \) is the number of observation above the threshold \( u \), we obtain;

\[
\hat{F}(x) = 1 - \frac{N_u}{n} \left(1 + \frac{\xi(x-u)}{\beta} \right)^{-\frac{1}{\xi}}
\]

(10)

Inverting (10) for a given probability \( \alpha \) and \( \alpha > F(u) \) gives VaR\(\alpha\)(\(X\));

\[
\hat{\text{VaR}}_{\alpha}(X) = q_{\alpha}(F) = u + \frac{\beta}{\xi} \left[ \frac{n}{N_u}(1-\alpha) \right]^{-\frac{1}{\xi}} - 1
\]

(11)

The associated expected shortfall at \( \hat{\text{VaR}}_{\alpha}(X) > u \), can be calculated as,

\[
\hat{\text{ES}}_{\alpha}(X) = \frac{\hat{\text{VaR}}_{\alpha}(X)}{1-\xi} + \frac{\beta - \xi u}{1-\xi}
\]

(12)

3. Methodology

This section describes the methodology applied to estimate GPD parameters and its threshold selection. Under the two-step procedure, we also describe the models used in volatility prediction and the corresponding innovation distribution. Finally, the backtesting procedure will be presented.

3.1. Estimating GPD Parameters

Given the data \( \tilde{X}_1, ..., \tilde{X}_{N_u} \) be the loss exceeding threshold \( u \) where \( N_u \) is the number of observations beyond threshold. For each of exceedances we calculate \( Y_j = \tilde{X}_j - u \). Then we estimate the GPD parameters by fitting this distribution to the \( N_u \) excess losses. The corresponding log-likelihood function can be represented as,

---

\[ ^{\dagger} \text{A natural estimator for } F(u) \text{ is given by empirical distribution.} \]
\[
L(\xi, \beta | \tilde{X}) = \begin{cases} 
- N_u \ln(\beta) - \left( \frac{1}{\xi + 1} \right) \sum_{j=1}^{N_u} \left[ 1 + \frac{\xi}{\beta} (\tilde{X}_j - u) \right], & \text{if } \xi \neq 0 \\
- N_u \ln(\beta) - \left( \frac{1}{\beta} \right) \sum_{j=1}^{N_u} (\tilde{X}_j - u), & \text{if } \xi = 0
\end{cases}
\]

The above log-likelihood function must be maximized subject to the parameter constraints that \( \beta > 0 \) and \( 1 + \xi (\tilde{X}_j - u) / \beta > 0 \) for all \( j \). The estimation of \( \xi \) and \( \beta \) can be computed using MATLAB software. VaR and ES may be directly read in the plot or calculated from equations (11) and (12) by replacing with our estimated parameters.

The crucial step in estimating GPD parameters is the determination of threshold \( u \). The choice of \( u \) ultimately involves the tradeoff between bias and variance. If the threshold is conservatively selected with few order statistics in the tail, then the tail estimate will be sensitive to outliers in the distribution and have a higher variance. On the other hand, to extend the tail more into the central part of the distribution creates a more stable index but results in a biased value. This sensitive tradeoff can be dealt with a variety of ways but there is no standard methodology of selecting the right threshold. As suggested by McNeil and Frey (2000), we set the constant \( N_u \) to be the 90th percentile of the innovation distribution or \( N_u = 0.1n \), where \( n \) is the rolling window size which is equal to 1,000 observations or approximately four years length for prediction. With this procedure, it actually fixes the number of index return data in the tail to be 100 i.e. use the largest 10% of the realized losses as a threshold for historical rolling window. This effectively give us a random threshold at the \((N_u+1)\)th order statistic.

### 3.2. Conditional Risk Measurement

Let \( X_t \) be a strictly stationary time series representing daily observations of the negative log return\(^3\). Assuming that the dynamics of \( X \) are given by,

\[ X_t = \sigma_t Z_t, \]

where the innovations \( Z_t \) are a strict white noise process (i.e. i.i.d.) with zero mean, a unit variance and distribution function \( F_Z(z) \). It is assumed that mean of \( X_t \) is zero and \( \sigma_t \) are measurable with respect to the given information of the return available up to time \( t-1 \) \((\mathfrak{R}_{t-1})\).

Given \( F_X(x) \) as the distribution of \( X_t \) and \( F_{X_{t+1}|\mathfrak{R}_t}(x) \) denotes the predictive distribution of the return over the next day, given the knowledge up to day \( t \). For \( 0 < \alpha < 1 \), an unconditional quantile and unconditional ES can be exhibited as equation (1) and (2) while the conditional quantile which is the predictive distribution for the return over the next day can be expressed as follows;

\[
VaR'_{\alpha} = \inf \left\{ x \bigg| F_{X_{t+1}|\mathfrak{R}_t}(x) \geq \alpha \right\}
\]

And the conditional expected shortfall would be,

\[
ES'_{\alpha} = E \left[ X_{t+1} \bigg| X_{t+1} \geq VaR'_{\alpha}, \mathfrak{R}_t \right]
\]

---

\(^3\) For ease of computation, we transform losses into positive numbers.
Since \( F_{Z_i|\mu_i,\sigma_i}(x) = P\{\mu_i + \sigma_i Z_i \leq x|\mu_i\} = F_x\left(\frac{x - \mu_i}{\sigma_i}\right) \), the above equations are simplified to,

Equation (17) is the calculation for risk measures for conditional one-period loss distribution where \( z_\alpha \) is the \( \alpha^{th} \) quantile of the distribution of \( Z \). Regarding to equation (17), we require estimates of \( \sigma_{t+1} \), the conditional volatility of the process and the quantile of the innovation distribution function \( F_Z \) which we explain in the coming section.

3.2.1. Volatility Forecasting

In this section, we consider GARCH and EWMA as the models for conditional volatility prediction. These two models are compared with the benchmark volatility which is simply the equally weight volatility obtained from the following formula,

\[
\hat{\sigma}_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} X_i^2
\]  

GARCH volatility prediction

Suppose that the data \( X_{t-n+1}, ..., X_t \) follow a particular model in GARCH family. We want to forecast the next day volatility \( \sigma_{t+1} \). This is closely related to the problem of predicting \( X_{t+1}^2 \). Assuming that we have access to the time history of the process up to time \( t \), \( \mathcal{R}_t = \sigma\left(X_s \right| s \leq t \). The conditional volatility prediction under GARCH(\( p,q \)) process can be obtained as follows,

\[
E\left(X_{t+1}^2 | \mathcal{R}_t\right) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2
\]  

In practice, low order GARCH models are most widely used. Consequently we apply GARCH(1,1) throughout this paper. Following equation (22) we have a GARCH(1,1) process,

\[
\hat{\sigma}_{t+1}^2 = \hat{E}\left(X_{t+1}^2 | \mathcal{R}_t\right) = \alpha_0 + \alpha_1 X_t^2 + \beta_1 \hat{\sigma}_t^2
\]  

where \( \alpha_0 > 0, \alpha_1 > 0, \beta_1 > 0 \), strictly stationary is ensure by \( \alpha_1 + \beta_1 < 1 \). To estimating \( \sigma_{t+1} \) by fitting GARCH(1,1) we follow McNeil and Frey (2000) by applying pseudo maximum likelihood (PML) approach.

Exponential Smoothing for Volatility

In traditional risk management application the volatility is forecasted by using the exponentially weighted moving average (EWMA) as used in RiskMetrics. Following this technique, the volatility prediction is,

\[
\sigma_{t+1}^2 = (1-\lambda) \sum_{i=1}^{n} \lambda^{i-1} X_{t-i+1}^2
\]  

For a high number of \( n \), we define a recursive scheme for one-step-ahead volatility forecasting as,

\[
\hat{\sigma}_{t+1}^2 = (1-\lambda) X_t^2 + \lambda \hat{\sigma}_t^2
\]  

7
where \( \lambda \) is the decay factor\(^4\). To estimate the relevant parameter \( \lambda \), the corresponding minimized root mean square error (RMSE) can be obtained using an iterative optimization procedure.

### 3.2.2. Estimating Innovation Distribution

Standard residuals, \( z \) can be calculated accordingly after we compute \( \hat{\sigma}_{t-a+1}, ..., \hat{\sigma}_{t} \) recursively from equation (18), (20) or (22) depending on the selected volatility approach. The model innovations are now calculated,

\[
(z_{t-a+1}, ..., z_{t}) = \left( \frac{X_{t-a+1}}{\hat{\sigma}_{t-a+1}}, ..., \frac{X_{t}}{\hat{\sigma}_{t-1}} \right)
\]

(23)

Under EVT framework, the procedure in estimating the particular standardized residual distribution is resemble to what described in section 2.2.3 and 3.1. The innovations \( (z_{t-a+1}, ..., z_{t}, z_{i}) \) can be defined as the order statistics \( z_{(i)} \leq z_{(2)} \leq ... \leq z_{(n)} \). We fix the number of data in the tail to be \( N_{a}=k \) and \( k \ll n \). It gives us the effectively random threshold at the \((k+1)\)th order statistic. The GPD with parameters \( \xi \) and \( \beta \) is fitted to the data \( \left( z_{(i)} - z_{(k+1)}, ..., z_{(k)} - z_{(k+1)} \right) \) which is the excess over the threshold for all residuals above the threshold. The form of the tail estimator can be written as,

\[
F_{i}(z) = 1 - \frac{k}{n} \left[ 1 + \xi \frac{z - z_{(k+1)}}{\beta} \right]^{-\xi}
\]

(24)

For probability \( \alpha > 1-k/n \), we can invert the tail formula and get

\[
\hat{z}_{a} = z_{a,k} = z_{(k+1)} + \frac{\beta}{\xi} \left[ \frac{n}{k} (1 - \alpha) \right]^{-\xi} - 1
\]

(25)

The estimated innovation can be plugged-in equation (17) obtaining VaR estimated from conditional EVT framework. According to McNeil and Frey (2000), we obtain the corresponding conditional ES estimate,

\[
ES_{\alpha}^{t} = \frac{1}{1 - \hat{\xi}} + \frac{\hat{\beta} - \hat{\xi} z_{(k+1)}}{(1 - \hat{\xi}) \hat{z}_{a}}
\]

(26)

Apart to Gaussian innovations, the standardized residual distribution is estimated easily as previously discussed in equation (3) and (4) but we have to change to the appropriate volatility estimation. Under the historical innovations, we use filtered historical simulation (FHS) technique proposed by Christoffersen (2006) that the residual, \( \hat{\xi} \) will be estimated through the standardized historical data.

As a result, we obtain nine methods in estimating VaR and ES which can be categorized as following,

- Gaussian, Gaussian-GARCH, Gaussian-EWMA (RiskMetrics)
- HS, HS- GARCH, HS- EWMA
- EVT, EVT- GARCH, EVT- EWMA

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\(^4\) The decay factor used in RiskMetrics is set to 0.94 for daily VaR estimation.
3.3. Evaluating VaR and ES Performance

To investigate the performance among various VaR and ES models with given rolling window size of 1,000, we propose the backtesting techniques. To illustrate, a window size of 1,000 is placed between the 1st and the 1000th data points, each model is estimated and the forecast is obtained for the 1001st day at different quantiles. Next, the window is moved one period ahead to 2nd and 1001st data points to obtain the forecast of the 1002nd day return with updated parameters from this new sample.

3.3.1. Evaluating VaR performance and Hypothesis testing

The relative performance of VaR is summarized by a violation ratio. A violation occurs if the realized return is greater than the estimation in a given day. When we forecast VaR at a certain quantile level \( \alpha \), it is expected that \( 1 - \alpha \) percent of the time the realized loss will be greater than the estimated VaR if the model performs well. Therefore, in theory, the violation ratio of an effective model should be \( 1 - \alpha \). If it is larger than \( 1 - \alpha \), the model underestimates. Conversely, the model overestimates the risk if violation ratio is less than \( 1 - \alpha \).

To assess the quality of VaR prediction, we compare \( X_{t+1} \) with \( \text{VaR}_\alpha \) for confidence level \( \alpha \in \{0.95, 0.99\} \) and investigate whether the failure rate of each model equal to the expected violations ratio \((1 - \alpha)\). From Timotheos and Degiannakis, (2006), the null and the alternative hypotheses should be,

\[
H_0 : N/T = 1 - \alpha \\
H_1 : N/T \neq 1 - \alpha,
\]

where \( N \) is number of exceptions and \( T \) is total number of observations, hence \( N/T \) is the proportion of excessive losses or violation ratio. The appropriate likelihood ratio statistics is,

\[
LR_{\text{uc}} = -2 \ln \left( \alpha^{T-N} (1-\alpha)^N \right) + 2 \ln \left( 1 - \frac{N}{T} \right)^T \left( \frac{N}{T} \right)^N \sim \chi^2_1 \tag{27}
\]

This is known as Kupiec’s test. Automatically, the log likelihood ratio is chi-square distributed with one degree of freedom. This test can reject a model for both high and low failures. When p-value is less than 0.05, we reject the null hypothesis.

Unlike the unconditional coverage Kupiec’s Test, Christoffersen has developed the conditional coverage test which jointly examines the conjecture that the total number of violations is statistically equal to the expected one and VaR violations are independent. The hypotheses are presented in the following equation,

\[
H_0 : N/T = 1 - \alpha, \pi_{01} = \pi_{11} = 1 - \alpha \\
H_1 : N/T \neq 1 - \alpha, \pi_{01} \neq \pi_{11} \neq 1 - \alpha,
\]

The corresponding likelihood ratio to the null hypothesis that the violations process is independent and the expected proportion of violations is equal to \((1 - \alpha)\) is:

\[
LR_{\text{cc}} = -2 \ln \left( \alpha^{T-N} (1-\alpha)^N \right) + 2 \ln \left( 1 - \pi_n \right)^{\pi_{00}} \left( 1 - \pi_n \right)^{\pi_{10}} \left( 1 - \pi_n \right)^{\pi_{01}} \left( 1 - \pi_n \right)^{\pi_{11}} \sim \chi^2_2 \tag{28}
\]

where \( n_{ij} \) is number of observations with value \( i \) followed by \( j \), for \( i, j = 0, 1 \) respectively and \( \pi_i = n_{ij}/\sum_j n_{ij} \) is the corresponding probabilities. \( i, j = 1 \) that an exception has been made, while \( i, j = 0 \) indicates the opposite. If the sequence of violation is independent, the probabilities to observe a VaR violation or not in the next period must be equal, or \( \pi_{01} = \pi_{11} = 1 - \alpha \). We reject the null hypothesis when p-value is less than 0.05.
3.3.2. Evaluating ES performance and Hypothesis testing

Following McNeil and Frey (2000), we conduct hypothesis testing on ES dealing with the size of discrepancy between \( X_{t+1} \) and \( ES_{\alpha}^t \) when violation occurs. We define residual as,

\[
\{ r_{\alpha,t} : t \in T, x_{\alpha,t} > \text{VaR}_{\alpha}^t \}, \text{where } r_{\alpha,t} = \frac{x_{\alpha,t} - \hat{ES}_{\alpha}^t}{\hat{\sigma}_{\alpha,t}}
\]  

(29)

where \( \hat{ES}_{\alpha}^t \) is an estimated expected shortfall. For the particular case where the dynamics of volatility and the first moment of the innovation distribution is correctly specified, the residuals should be i.i.d. and conditional on \( \{ X_{\alpha,t} > \text{VaR}_{\alpha}^t \} \), they have expected value of zero. Under the null hypothesis that the expected value of discrepancies should be equal to zero, we employ the bootstrap test using algorithm from Efron and Tibshirani (1993, p224). In this case, we use one-side test against alternative hypothesis that the residuals have mean greater than zero or, equivalently, the expected shortfall is systematically underestimated. We reject the model when the p-value is less than 0.05.

4. Descriptive Statistics of Daily returns

It should be noted that we use daily return data so both conditional and unconditional VaR and ES estimation are made on daily basis. VaR and ES calculation is based on the realized losses (left-tail) within given historical window size of 1,000 days and it is moved one day forward (rolling widow). The estimation is conducted at 95% and 99% confidence level respectively.

We conduct the study through the countries in MSCI All Country Far East Index (MSCI AC Far East\(^5\)) including Japan to investigate VaR and ES performance as well as to validate the accuracy of our approach. Our datasets consists of ten MSCI Country Indices which are Hong Kong, Japan, Singapore, China, Indonesia, Korea, Malaysia, Philippines, Taiwan and Thailand. The study covers the period of January 1, 1993 to December 31, 2007. Throughout this paper, we work with the natural logarithm of daily returns, using daily closing prices.

\[
X_{i,t} = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right)
\]

(30)

where \( P_{i,t} \) is the daily closing value of the index \( i \) on day \( t \) and \( X_{i,t} \) is the return of index \( i \) on day \( t \).

Basic statistics for daily return series is shown in Table I. According to the Jarque-Bera statistics whose test is a goodness-of-fit measure of departure from normality based on the sample kurtosis and skewness, all countries strongly reject the assumption of normality. In depth, the kurtosis of all the datasets are greater than three\(^6\) indicating the evidence of fat-tailed.

Regarding to the standard deviation, Korea and Thailand are most volatile. However, Malaysia exhibits the lowest and highest daily returns during the period 1993-2007 following by Thailand and Indonesia respectively. The empirical returns pattern of each Asian index can be seen in Figure 1.

---

\(^5\) The description of MSCI index categories can be found in http://www.msci.com/equity/indexdesc.html/#WORLD.

\(^6\) Normal distribution has a kurtosis of three and skewness of zero. If the skewness is greater than three regardless to their mean or standard deviation, it represents the leptokurtic or fat-tailed distribution.
A better characterization of fat-tailed distribution of each index returns can be seen in the QQ-plot in Figure 2. In this case, quantiles of an empirical index return distribution are plotted against the standard normal quantiles. If the distribution of the returns is normal, the QQ-plot should close to linear. However, our empirical data shows that there exist the deviations from normality. In this case, the plot curve upward at the left and downward at the right which is the evidence of heavy tails distribution rather than normal distribution. As a result, it is sensible to estimate VaR and ES using EVT technique, in turn, the generalized Pareto distribution.

5. Empirical Results

In this section, we perform the volatility forecasting, the estimation of VaR and ES and the backtesting result respectively.

5.1. Volatility Prediction

We estimate volatility of each index returns with different methods namely, equally weight (unconditional model), EWMA and GARCH(1,1). Figure 3 illustrates an arbitrary 1,000-day excerpt from negative log returns on MSCI Hong Kong Index and MSCI Thailand Index which including the period of East Asian financial crisis during the year 1997.

The volatility prediction exhibits that the conditional volatility models can quickly respond to market change than unconditional volatility. The evidence in Hong Kong shows that EWMA volatility is the most quickly responded to the dramatically change of the market while GARCH volatility performs quite similarly to the 0.94-EWMA volatility model. In case of Thailand, however, it demonstrates that most of conditional volatility models are trivially different. In depth, GARCH model is slightly the most quickly reacted to the market change following by EWMA and 0.94-EWMA respectively. Like Thailand, most of countries also indicate the speedy response to market movement of GARCH volatility better than others.

It is noticed that EWMA model can capture the dramatically change in market index return better than GARCH model as evidence in Hong Kong. However, GARCH model provides much more speedy and flexible adjustment in volatility prediction that the adjustment of EWMA model is smoother than GARCH as in the case of Thailand.

5.2. VaR and ES performance

We backtest the nine models of VaR and ES estimation on ten historical series of Asian Index log returns starting from November 1, 1996 to December 31, 2007 or totally 2,912 observations. Figure 4 illustrates nine models used in VaR estimation on MSCI Thailand Index and the corresponding historical loss. Three main models: Gaussian, HS and GPD are

\(^7\) This refers to the EWMA with decay factor of 0.94 as conventionally used in RiskMetics.
\(^8\) The volatility prediction of the remaining countries can be found in figure A1 in Appendix section.
illustrated separately with the three volatility models; Simple, GARCH and EWMA respectively.

[ Figure 4 is here ]

The figure shows that VaR estimated under unconditional models are the most conservative as it provides the smooth and stable VaR pattern comparing to conditional ones that better respond to the change of the index return. The estimate VaR based on HS and GPD approach are graphically the same. Particularly, it is interesting to demonstrate the VaR performance on December 19, 2006 or “Black Tuesday”. The result shows that all of Gaussian-based models cannot capture the extreme loss occurred. This is similar to the cases of unconditional HS and unconditional GPD. Within conditional models, the EWMA-based models both embedded in HS approach and GPD approach can capture the extreme event although it tend to overestimate VaR while GARCH-based models provide underestimation.

To test the validity and accuracy of overall period, we calculate VaR violation of each model as exhibited in Table II while the corresponding p-values from Kupiec’s test are described in Table III. As a decision rule, we interpret a p-value less than or equal to 0.05 as evidence against the null hypothesis.

[ Table II and III are here ]

The result in Table III presents that in almost all countries, violations are closest to the expected ratio at 95% confidence level (80 cases out of 90 cases). The FHS and the conditional GPD perform better than the unconditional models since they are accepted in all countries. However, the unconditional models still work well specifically those with non-normality assumption. Although the Gaussian-based models cannot be rejected at 95% confidence level, they perform less accurately against others. Besides, the estimates VaR under normality assumption (Gaussian VaR) tend to underestimate at a higher confidence level as its VaR violation ratio is almost two times higher than the expected while the corresponding Kupiec’s test is extremely rejected in almost all Gaussian VaR at 99% confidence level. Unlike Gaussian models, HS and GPD perform well in all given confidence levels. Specifically our result in GARCH-based models is consistent with McNeil and Frey (2000) that the GPD-GARCH VaR does not fail in all cases and outperforms Gaussian-based models. The result is similar to those conditioned with EWMA volatility. Like conditional GPD, the same pattern occurs in FHS approach.

It can be inferred that the Gaussian VaR models provide less consistency and less reliability compared to the non-normality models. Unfortunately, we cannot distinguish VaR estimation based on both HS and GPD models as they perform insignificantly different in almost all cases.

[ Table IV is here ]

Table IV presents the other backtesting procedure; the conditional coverage test namely Christoffersen’s test. This test is proposed to segregate the impact of VaR violation clustering. The result shows that all unconditional VaR models reveal the evidence of violation clustering and they are rejected in almost all countries. Among Gaussian models, Gaussian-GARCH performs the best at 95% confidence level but all Gaussian-based models tend to perform less accurately as they are almost all rejected at a further confidence level (rejected 27 cases out of 30 cases).
The conditional VaR estimates, both conditional GPD and FHS, remarkably outperform the unconditional ones because the models themselves typically incorporate the volatility clustering. In depth, there is no significantly evidence of the difference between EWMA-based and GARCH-based modeling under GPD approach or FHS approach. Even though GPD-EWMA slightly performs better than GPD-GARCH for the higher confidence level, it is opposite to FHS. Comparing conditional GPD with FHS, the conditional VaR estimated by GPD is not much different from FHS especially at the lower confidence level although it is trivially better in higher confidence level particularly the GPD-EWMA approach.

According to McNeil and Frey (2000), we conduct the one-side bootstrap test for ES estimation. Figure 5 depicts the exceedance residuals of MSCI China Index at 95% confidence level. The left graphs show clear evidence against the conditional normality assumption; the right ones are conditional GPD and FHS respectively. The figure indicates that the residuals derived from Gaussian assumption may underestimate as it has lower magnitude of negative residuals than those of GPD or historical simulation.

Table V proposes one-tailed test of the zero-mean residuals with the corresponding p-values derived from a bootstrap test. It shows that the residuals derived from Gaussian assumptions usually fail the test with p-value in almost all case less than 0.05. As a result, we conclude that an assumption of conditional Gaussian is not powerful enough to predict ES. The result is consistent with the study of McNeil and Frey (2000).

The hypothesis of zero-mean exceedance residuals is quite strongly supported for all non-normality models in almost all countries. Regarding to the unconditional non-normality models, even the unconditional GPD perform indifferent from HS, it is slightly superior to HS in a high confidence level. Comparing to conditional models, conditional GPD and FHS residuals have more plausibly mean zero than the unconditional models at lower confidence level but it is not much different for the higher confidence level. This is because those models can take into account the leptokurtosis of the tail distribution of index losses.

If we look deeply through the conditional ES, GPD-EWMA has better performance than GPD-GARCH. The superiority of EWMA based model is clearer when using FHS in ES estimation. However, it is not decisive in the superiority of conditional GPD and FHS approaches as they perform trivially different in all given confidence level.

6. Conclusion

The study focuses on the estimation of Value-at-Risk (VaR) and expected shortfall (ES). To account for stochastic volatility, we fit the conditional volatility based on GARCH and EWMA approaches with the GPD estimation suggested by extreme value theory to model the distribution of the tail. Our conditional approaches are designed based on the two-step procedure as suggested by McNeil and Frey (2000). Various models are compared with traditional methods of tail estimation which are Gaussian approach and historical simulation (HS) approach.

Our finding indicates that the conditional approaches are preferable to the unconditional both in VaR and ES measurement. Specifically, the models with Gaussian assumption usually underestimate as they fail to capture the leptokurtosis which is often observed in financial returns. The result is consistent with that of McNeil and Frey (2000). When we compare with the non-parametric model such as HS and its modification; filtered historical simulation (FHS), both unconditional GPD and simple HS perform less accurately in VaR estimation especially for higher confidence level. However, under ES estimation, unconditional GPD
and HS perform quite similarly to those conditional models. It is remarkable that the conditional GPD-based models perform trivially different from FHS both in VaR and ES estimation.

Regarding the stochastic volatility incorporated in each tail estimation models, we extend McNeil and Frey (2000) procedure by proposing the simpler conditional volatility forecasting namely exponential weighted moving average (EWMA) rather than using only GARCH-based volatility model. Our result shows that model with more complexity does not always outperform the model with less complexity. While GARCH can reflect more flexible adjustment than EWMA, our study indicates that the EWMA-based models are still useful in both VaR and ES estimation as it has a simpler algorithm and perform quite similarly (or slightly better) to those of GARCH-based modeling. However, the difference between these two conditional volatility models may occur if the holding period is longer since GARCH models generally play an important role in longer memory according to a more flexible structure. But the detailed analysis is left for future study.
References


Table I

Descriptive Statistics of Daily Returns for Ten MSCI Far East Country Indices

This table reports the descriptive statistics of the daily returns of ten Asian MSCI Country Indices: Hong Kong, Japan, Singapore, China, Indonesia, Korea, Malaysia, Philippines, Taiwan and Thailand. The total number of observations is 3,912 days ranging from January 1, 1993 to December 31, 2007 (excluding holidays). The Jarque-Bera test of normality is performed with the corresponding p-value at the lower adjacent row.

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<td>3912</td>
<td>3912</td>
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Table II

**VaR Violation Ratio**

This table presents expected violation ratio and the corresponding violation ratio defined at 95% and 99% confidence level of ten Asian MSCI Country Indices. The violations are obtained from nine models: Gaussian, Gaussian-GARCH, Gaussian-EWMA (RiskMetrics), historical simulation (HS), filtered historical simulation (FHS) with GARCH and EWMA, unconditional GPD, GPD-GARCH and GPD-EWMA. The higher the ratio than the expected violation indicates the higher chance of underestimate.

| Length of Test | Gaussian | | | HS | | | GPD | | | EWMA | | | EWMA | | | EWMA |
|----------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|                 | Simple  | GARCH   | EWMA     | Simple  | GARCH   | EWMA     | Simple  | GARCH   | EWMA     | Simple  | GARCH   | EWMA     |
| Expected violation | 5.00%   | 5.00%   | 5.00%     | 5.00%   | 5.00%   | 5.00%     | 5.00%   | 5.00%   | 5.00%     | 5.00%   | 5.00%   | 5.00%     |
| Hong Kong       | 3.81%   | 4.88%   | 6.01%     | 4.53%   | 4.67%   | 5.12%     | 4.53%   | 4.57%   | 4.95%     |
| Japan           | 4.67%   | 4.91%   | 5.39%     | 5.36%   | 5.43%   | 5.15%     | 5.29%   | 5.49%   | 5.36%     |
| Singapore       | 4.98%   | 4.22%   | 4.95%     | 5.67%   | 4.74%   | 5.15%     | 5.70%   | 4.81%   | 5.01%     |
| China           | 5.39%   | 5.39%   | 6.15%     | 5.87%   | 5.43%   | 5.22%     | 5.67%   | 5.39%   | 5.43%     |
| Indonesia       | 5.12%   | 4.98%   | 6.11%     | 5.77%   | 5.25%   | 5.36%     | 5.60%   | 5.08%   | 5.53%     |
| Korea           | 5.29%   | 4.57%   | 5.70%     | 5.22%   | 4.40%   | 4.84%     | 5.05%   | 4.50%   | 4.81%     |
| Malaysia        | 3.85%   | 4.33%   | 5.22%     | 4.81%   | 4.43%   | 5.25%     | 4.74%   | 4.57%   | 5.12%     |
| Philippines     | 4.88%   | 4.16%   | 4.84%     | 5.25%   | 4.91%   | 5.15%     | 5.32%   | 4.77%   | 5.08%     |
| Taiwan          | 5.05%   | 5.46%   | 6.04%     | 4.57%   | 4.84%   | 4.88%     | 4.60%   | 4.74%   | 4.74%     |
| Thailand        | 4.36%   | 4.50%   | 5.98%     | 4.91%   | 4.50%   | 4.84%     | 4.84%   | 4.33%   | 4.84%     |

| Length of Test | Gaussian | | | HS | | | GPD | | | EWMA | | | EWMA | | | EWMA |
|----------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|                 | Simple  | GARCH   | EWMA     | Simple  | GARCH   | EWMA     | Simple  | GARCH   | EWMA     | Simple  | GARCH   | EWMA     |
| Expected violation | 1.00%   | 1.00%   | 1.00%     | 1.00%   | 1.00%   | 1.00%     | 1.00%   | 1.00%   | 1.00%     | 1.00%   | 1.00%   | 1.00%     |
| Hong Kong       | 1.65%   | 1.20%   | 1.75%     | 1.13%   | 0.93%   | 1.03%     | 1.13%   | 0.96%   | 1.00%     |
| Japan           | 1.75%   | 0.93%   | 1.13%     | 1.10%   | 0.93%   | 0.93%     | 1.03%   | 0.72%   | 0.82%     |
| Singapore       | 2.09%   | 1.41%   | 1.61%     | 1.34%   | 1.24%   | 0.96%     | 1.27%   | 1.10%   | 1.06%     |
| China           | 2.16%   | 1.48%   | 1.85%     | 1.37%   | 1.10%   | 1.13%     | 1.34%   | 1.06%   | 1.17%     |
| Indonesia       | 2.09%   | 1.79%   | 2.13%     | 1.37%   | 1.10%   | 1.24%     | 1.30%   | 1.06%   | 1.03%     |
| Korea           | 2.27%   | 1.37%   | 1.58%     | 1.44%   | 1.13%   | 1.06%     | 1.44%   | 0.79%   | 0.79%     |
| Malaysia        | 1.75%   | 1.51%   | 2.23%     | 1.06%   | 1.03%   | 1.34%     | 0.89%   | 0.93%   | 1.34%     |
| Philippines     | 1.72%   | 1.30%   | 1.72%     | 1.20%   | 1.06%   | 0.96%     | 1.03%   | 0.86%   | 0.93%     |
| Taiwan          | 1.61%   | 1.51%   | 1.85%     | 0.89%   | 1.00%   | 0.93%     | 0.82%   | 0.89%   | 1.03%     |
| Thailand        | 1.75%   | 1.58%   | 2.40%     | 1.10%   | 0.89%   | 1.17%     | 1.03%   | 0.86%   | 0.96%     |
Table III

VaR Backtesting Results (Unconditional Coverage Test)

This table presents VaR backtesting results for ten Asian MSCI Country Indices using Kupiec’s test (unconditional coverage test). The corresponding p-values are obtained from nine models: Gaussian, Gaussian-GARCH, Gaussian-EWMA (RiskMetrics), historical simulation (HS), filtered historical simulation (FHS) with GARCH and EWMA, unconditional GPD, GPD-GARCH and GPD-EWMA. Two quantile estimates are considered: 95% and 99% respectively. We reject the model if its p-value is less than or equal to 0.05.

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Table IV

VaR Backtesting Results (Conditional Coverage Test)

This table presents VaR backtesting results for ten Asian MSCI Country Indices using Christoffersen’s test (conditional coverage test). The corresponding p-values are obtained from nine models: Gaussian, Gaussian-GARCH, Gaussian-EWMA (RiskMetrics), historical simulation (HS), filtered historical simulation (FHS) with GARCH and EWMA, unconditional GPD, GPD-GARCH and GPD-EWMA. Two quantile estimates are considered: 95% and 99% respectively. We reject the model if p-value is less than or equal to 0.05.

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Table V

ES Backtesting Results (Bootstrap Test)

This table presents expected shortfall (ES) backtesting results for ten Asian MSCI Country Indices. The corresponding p-values for one-side bootstrap test of the hypothesis that the exceedance residuals have mean zero against the alternative that mean is greater than zero are obtained from 10,000 simulations within nine models; Gaussian, Gaussian-GARCH, Gaussian-EWMA (RiskMetrics), historical simulation (HS), filtered historical simulation (FHS) with GARCH and EWMA, unconditional GPD, GPD-GARCH and GPD-EWMA. Two confidence levels are considered: 95% and 99%. We reject the model if its p-value is less than or equal to 0.05.

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Figure 1

The Empirical Asian Index Returns Series

The figure plots the empirical returns of ten Asian MSCI Country Indices; Hong Kong, Japan, Singapore, China, Indonesia, Korea, Malaysia, Philippines, Taiwan and Thailand from January 1993 to December 31, 2007.
Figure 2

QQ-Plot of Each Index Returns versus Standard Normal
Figure 3

Empirical Losses and the Volatility Prediction

This figure depicts 1,000 daily losses (in positive numbers) of selected Asian MSCI Country Indices, Hong Kong and Thailand respectively with their corresponding volatility prediction from November 1996 to August 31, 2000. The upper plot illustrates the empirical losses. The middle plot demonstrates the estimate standard deviation from simple model (dashed line), EWMA with fixed decay factor 0.94 (solid line) and optimized EWMA (dotted line). The lower plot indicates standard deviation estimated by GARCH(1,1) model.
Figure 4

MSCI Thailand Index Losses and 95% VaR Estimates

The figure depicts 350-Day MSCI Thailand Index losses (in positive numbers) and 95% VaR estimates ranged from August 29, 2006 to December 31, 2007. The upper graph presents VaR estimated from Gaussian based models, the middle and the lower graphs present VaR estimated from historical simulation (HS) and GPD based on EVT framework respectively. The dashed-dotted line exhibits the unconditional method; the solid line and dotted line indicate GARCH-based and EWMA-based models respectively.
Figure 5

Exceedance Residuals for MSCI China Index Series with 95% Confidence Level

The figure presents exceedance residuals for MSCI China Index series with 95% confidence level. The upper panel illustrates (from left to right) residuals derived from Gaussian-EWMA (RiskMetrics), GPD-EWMA and HS-EWMA respectively. The lower panel illustrates (from left to right) residuals derived from Gaussian-GARCH, GPD-GARCH and HS-GARCH respectively.
Appendix

Figure A1

Empirical negative returns and the volatility prediction

This figure depicts 1,000 daily losses (in positive numbers) of MSCI Indices; Japan, Singapore, China, Indonesia, Korea, Malaysia, Philippines and Taiwan with their corresponding volatility prediction from November 1996 to August 31, 2000. The upper plot illustrates the empirical losses. The middle plot demonstrates the estimate standard deviation from simple (dashed line), EWMA with fixed decay factor 0.94 (solid line) and optimized EWMA (dotted line). The lower plot indicates standard deviation estimated by GARCH(1,1) model.
Figure A1 (Continue)

China

Indonesia
Figure A1 (Continue)

Korea

Malaysia
Figure A1 (Continue)

Philippines

Taiwan